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Innovative Applications of O.R.

Optimal relay node placement in delay constrained wireless sensor network design

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ABSTRACT

The Delay Constrained Relay Node Placement Problem (DCRNPP) frequently arises in the Wireless Sensor Network (WSN) design. In WSN, Sensor Nodes are placed across a target geographical region to detect relevant signals. These signals are communicated to a central location, known as the Base Station, for further processing. The DCRNPP aims to place the minimum number of additional Relay Nodes at a subset of Candidate Relay Node locations in such a manner that signals from various Sensor Nodes can be communicated to the Base Station within a pre-specified delay bound. In this paper, we study the structure of the projection polyhedron of the problem and develop valid inequalities in form of the *node-cut* inequalities. We also derive conditions under which these inequalities are facet defining for the projection polyhedron. We formulate a branch-and-cut algorithm, based upon the projection formulation, to solve DCRNPP optimally. A Lagrangian relaxation based heuristic is used to generate a good initial solution for the problem that is used as an initial incumbent solution in the branch-and-cut approach. Computational results are reported on several randomly generated instances to demonstrate the efficacy of the proposed algorithm.

1. Introduction

A Wireless Sensor Network (WSN) consists of spatially distributed Sensor Nodes (SNs) to monitor physical or environmental conditions, such as temperature, pressure, motion or pollutants. These SNs transmit the sensed data through wireless communication to a Base Station (BS) (Clare, Pottie, & Agre, 1999). SNs may be placed inside the event to be monitored or in the proximity of the same. These features ensure a wide range of applications for WSN in varied areas, e.g. health-care, military operations, and environmental monitoring. WSN may be deployed in a vast geographical area (e.g. oceans, forests) in order to detect critical events such as forest-fire, tsunami, and floods. Using WSN, doctors can remotely monitor physiological condition of their patients. WSNs can be an essential part of military operations with their ability to perform key strategic tasks, e.g. battlefield surveillance, reconnaissance of rival armies etc. (cf. Akyildiz, Su, & Sankarasubramaniam, 2002).

Transmission radius of SNs (the range beyond which they cannot transmit the signals) is typically several tens of meters. Due to the limited transmission radius and the vastness of target geographical region, usually a multi-hop wireless communication, using some additional Relay Nodes (RNs), is required to facilitate end-to-end communication between SNs and the BS. As the cost of RNs ranges from tens to hundreds of dollars, minimizing the number of additional RNs without compromising the quality of signals is an important aspect of the WSN design.

Objective of the Delay-Constrained Relay Node Placement Problem (DCRNPP) is to design a multi-hop wireless mesh network with minimum number of additional RNs in order to facilitate wireless-communication between each of the SN and the BS. The placement of RNs should ensure that the delay on the paths between BS and the SNs is restricted within a pre-specified delay bound. DCRNPP studied in this paper is motivated by an important class of WSN, where locations of the Candidate Relay Nodes (CRNs) are known *a priori*. For example, in forest fire detection, a set of sites where the CRNs can be placed may be known beforehand. In brief, the key features of DCRNPP, studied in this paper, are the following.

- The locations of SNs and CRNs are known beforehand.
- Transmission radius of the SN/RN allows only certain links to be permitted in the graph.
- The objective is to obtain a sub-graph with minimum number of CRNs selected that connects all SNs to BS.
- Placement of RNs must ensure that there exists at least one path from each SN to the BS, for which the cumulative delay does not exceed a pre-specified delay bound Δ.

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The version of DCRNPP, with relaxation of the delay constraint, known as the Relay Node Placement Problem (RNPP) is broadly related to the Steiner tree problem (STP) in graphs (Gondran & Minoux, 1984). Classical STP is a NP-Hard problem and is extensively studied by various researchers (Robins & Zelikovsky, 2005; Karpensiki & Zelikovsky, 1993; Arora, 1998; Chopra & Rao, 1994a; Chopra & Rao, 1994b).

The Prize collecting Steiner tree Problem (PCSP) (or the Node weighted Steiner tree Problem, NSP), where node weights along with edge weights are specified, can be considered as a generalization of both STP and RNPP (Duin & Volgenant, 1987; Segev, 1987). Approximation algorithms for PCSP are proposed by various researchers (Klein & Ravi, 1995; Demaine & Hajiaghayi, 2009; Remy & Steger, 2009; Canuto, Resende, & Ribeiro, 2001; Klau et al., 2004). Fischetti (1991) studied the facial structure of a generalization of PCSP, known as the Steiner arborescence (or directed Steiner tree) problem, and pointed out that the PCSP can be transformed into it. Engevall, Lundgren, and Värbrand (1998) proposed another ILP formulation for the PCSP, based on the shortest spanning tree problem formulation, which was introduced originally by Beasley (1989) for the Steiner tree problem. A cutting plane algorithm for the PCSP based on generalized subtour elimination constraints was proposed by Lucena and Resende (2004).

The RNPP was studied as The Steiner Tree Problem with Minimum Number of Steiner Points and Bounded Edge Length (STP-MSPBEL) by Lin and Xue (1999). They showed the problem to be NP-complete and proposed a polynomial time 5-approximation algorithm for the problem. Cheng, Du, Wang, and Xu (2008) studied the same problem and proposed a 3-approximation and a 2.5-approximation algorithm. Voss (1999) studied the STP with hop constraints. The problem was shown to be NP-hard and a minimal spanning tree based heuristic was proposed to obtain a good feasible solution. Kim, Bang, and Choo (2006) studied the delay and delay variation constrained multicasting STP. The problem is similar to the one studied by Voss, with a delay constraint instead of the hop constraint, and a constraint on delay variation between two sources. They proposed a polynomial time heuristic algorithm for the problem. Costa, Cordeau, and Laporte (2008) studied the STP with revenue, budget, and hop constraints. They proposed a greedy heuristic for generating initial solution. The initial solution was improved by the destroy and repair or the tabu search algorithm. Gouveia, Paias, and Sharma (2008) studied rooted distance-constrained minimum spanning tree problem, and proposed a path based formulation. They presented a column generation scheme and a Lagrangian relaxation based approach combined with sub-gradient optimization procedure to solve the problem. Misra, Hong, Xue, and Tang (2008) studied the constrained relay placement problem for connectivity and survivability, and proposed an approximation algorithm for the same. Their model took into account the transmission radius as the edge length bound. Bhattacharya and Kumar (2010) studied DCRNPP and showed the problem to be NP-Hard. They presented a local search based greedy heuristic to provide an approximate solution for the problem.

Another class of problem closely related to the RNPP is the Network Design Problem with Relays (NDPR). The NDPR is defined on an undirected graph $G = (V,E,K)$, where V and E are the vertex and edge sets, respectively. Set $K = \{o(k) \in V, d(k) \in V\}$ is a set of communication pairs or commodities. Here, $o(k)$ and $d(k)$ denote the origin and destination of kth commodity, respectively. A cost c_{ij} is associated with each edge $(i,j) \in E$ and a fixed cost f_i, of installing a relay at vertex i, is associated with each vertex $i \in V$. The objective of NDPR is to select a subset $E' \subseteq E$ and a subset $V' \subseteq V$ in such a way that the total cost of network (edge cost and the relay installation cost) is minimized, and there exists a path linking the origin $o(k)$ and destination $d(k)$ for each commodity $k \in K$ in which the length between any two consecutive nodes does not exceed a preset upper bound. Cabral, Erkut, Laporte, and Patterson (2007) developed a lower bound procedure and several heuristics for NDPR. They compared these algorithms on several randomly generated test instances. Li, Aneja, and Huo (2011) developed a Branch-and-Price algorithm for directed version of NDPR, using an arc-path formulation. Konak (2012) presented a new formulation for NDPR based on set covering constraints. Using the new formulation, he proposed a Genetic Algorithm based heuristic to solve NDPR.

To the best of our knowledge, there is no algorithm in the literature that solves DCRNPP optimally. Apart from developing a branch and cut based exact algorithm to solve DCRNPP, this paper also examines the polyhedron structure of the problem and proposes a projection formulation for DCRNPP. With the help of computational experiments on several randomly generated test instances, we demonstrate that the proposed algorithm based upon projection formulation is able to optimally solve problem instances of size up to 50 SNs and 200 CRNs, within reasonable CPU time.

The paper is organized as follows. In Section 2, we describe a mathematical formulation for DCRNPP that involves an exponentially large number of path variables. Since not all the columns in this formulation are known explicitly, a column generation approach is presented in Section 3. The column generation approach solves the LP relaxation of the path-based formulation to compute a valid lower bound for the optimal solution. In Section 4, we describe a projection formulation for the problem, which involves variables corresponding to CRNs only. We identify a set of valid inequalities for the projection formulation, known as the *node cut inequalities*. These inequalities are facet defining for the projection polyhedron under certain mild conditions. In Section 5, we present two separation algorithms to generate violated node cut inequalities for the projection formulation. A heuristic to generate a good feasible solution, which is used as an initial incumbent in the branch and cut algorithm, is discussed in Section 6. In Section 7, we discuss the implementation details for the branch and cut algorithm that is used to solve DCRNPP optimally. Computational results on various randomly generated test instances are reported in Section 8. In Section 9, we summarize this work and identify possible areas for future research.

2. Problem formulation

In this section, we describe a path-based formulation for the DCRNPP. The problem is defined on an undirected graph $G = (V,E)$. The SN set $T = \{1,2,\ldots,m\}$, CRN set $R = \{1,2,\ldots,n\}$ and the BS (node 0) constitute the node set of the graph i.e. $V = T \cup R \cup \{0\}$. The edge set in the graph is defined as $E = \{(i \in V, j \in V): l_{ij} \leqslant r\}$, where l_{ij} is the Euclidian distance, between node i and node j, bounded by the transmission radius r. A non-negative delay d_{ij} is associated with each edge $(i,j) \in E$ and may be defined as some function of the Euclidean distances l_{ij}. We define the set of neighbors for each node $i \in V$, as $\mathcal{N}_i = \{j \in V : (i,j) \in E\}$.

The set P^k is defined as set of all paths between BS and the SN $k \in T$. Set $P(= \bigcup_{k \in T} P^k)$ contains all paths between BS and all SNs. The set $P^{k,\Delta}(\subseteq P^k)$ is defined as set of all paths between the BS and SN $k \in T$ within the delay bound Δ. Set P^{Δ} consists of all delay-constrained paths between the BS and all SNs, i.e. $P^{\Delta} = \bigcup_{k \in T} P^{k,\Delta}$.

Following variables are used in the problem-formulation:

- For each CRN $j \in R$, binary variable y_j indicates whether a RN is placed at CRN location j or not.