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Discrete Optimization

Branch-and-price-and-cut for the multiple traveling repairman problem with distance constraints

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ABSTRACT

In this paper, we extend the multiple traveling repairman problem by considering a limitation on the total distance that a vehicle can travel; the resulting problem is called the *multiple traveling repairmen problem with distance constraints* (MTRPD). In the MTRPD, a fleet of identical vehicles is dispatched to serve a set of customers. Each vehicle that starts from and ends at the depot is not allowed to travel a distance longer than a predetermined limit and each customer must be visited exactly once. The objective is to minimize the total waiting time of all customers after the vehicles leave the depot. To optimally solve the MTRPD, we propose a new exact branch-and-price-and-cut algorithm, where the column generation pricing subproblem is a resource-constrained elementary shortest-path problem with cumulative costs. An ad hoc label-setting algorithm armed with bidirectional search strategy is developed to solve the pricing subproblem. Computational results show the effectiveness of the proposed method. The optimal solutions to 179 out of 180 test instances are reported in this paper. Our computational results serve as benchmarks for future researchers on the problem.

1. Introduction

The traveling repairman problem (TRP) has been extensively studied by a large number of researchers (e.g., Afrati, Cosmadakis, Papadimitrious, Papageorgiou, & Papakostantinou (1986), García, Jodrá, & Tejel (2002), and Salehipour, Sörensen, Goos, & Bräysy (2011)); this problem is also termed the minimum latency problem (Arora & Karakostas, 2003; Archer, Levin, & Williamson, 2008; Blum et al., 1994), the traveling deliveryman problem (Fischetti, Laporte, & Martello, 1993; Méndez-Díaz, Zabala, & Lucena, 2008; Minieka, 1989) and the cumulative traveling salesman problem (Bianco, Mingozzi, & Ricciardelli, 1993). The TRP is defined on a complete graph $G = (V, E)$, where $V = \{0, 1, \ldots, n, n+1\}$ is the vertex set and $E = \{(i, j)\colon i, j \in V, i \neq j, i \neq n+1, j \neq 0\}$ is the edge set. Vertices 0 and $n+1$ represent the exit from and the entrance to the depot, respectively. We denote the vertices representing the set of n customers by $V_C = \{1, \ldots, n\}$. The repairman (henceforth referred to as vehicle) is assumed to travel at a constant speed. Each edge (i, j) has a non-negative length $d_{i,j}$ and requires a non-negative traversing time $t_{i,j}$, which is symmetric, i.e., $t_{i,j} = t_{j,i}$, and satisfies the triangle inequality rule. The objective of the TRP is to find a Hamiltonian tour on G, starting from vertex 0 and ending at vertex $n+1$, which minimizes $\sum_{i \in V_C} l_i$, where l_i denotes the waiting time of customer i after the vehicle leaves vertex 0. A direct generalization of the TRP is the multiple traveling repairman problem (MTRP) that considers K identical vehicles (Fakcharoenphol, Harrelson, & Rao, 2007). Applications of the TRP and MTRP can be found in routing pizza deliverymen, routing automated guided vehicles through cells in a flexible manufacturing system or scheduling machines to minimize mean flow time for jobs (Fischetti et al., 1993).

This paper studies an extension of the MTRP by involving a distance constraint that the route length (or duration) of each vehicle cannot exceed a predetermined limit L. This type of constraint usually stems from regulations on working hours for workers or arises in the home delivery of perishable products. We call the resulting problem the *multiple traveling repairman problem with distance constraints* (MTRPD) whose objective is to find K routes such that each vertex is visited exactly once, the distance constraint is respected and the total waiting time of all customers is minimized. Examples of other vehicle routing models that incorporate the distance constraint can be found in Laporte, Nobert, and Desrochers (1985), Li, Simchi-Levi, and Desrochers (1992), and Erera, Morales, and Savelsbergh (2010).

The MTRP can be viewed as a variant of the *multiple traveling salesman problem* (m-TSP) (Bektas, 2006; Svestka & Huckfeldt,

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1973). Although many researchers have studied the TRP, the literature on the MTRP is surprisingly limited. The only prior study we can find in existing literature is Fakcharoenphol et al. (2007). They presented a polynomial-time 8.497γ-approximation algorithm for the MTRP, where γ denotes the best polynomial-time approximation factor possible for the k minimum spanning tree (k-MST) problem (Arora & Karakostas, 2006).

Apart from the MTRPD, there exist several other extensions of the MTRP in the literature. Bennett and Gazis (1972) introduced a *school bus routing problem* (SBRP) in which a fleet of school buses is dispatched to take pupils from pick-up points to school. Each bus has a fixed capacity and each pick-up point has a given demand, represented by the number of pupils. The objective of this problem is to minimize the weighted sum of the total bus travel time and the total pupil travel time. Li and Fu (2002) described a case study of routing school bus for Hong Kong kindergartens. They formulated the problem as a multi-objective combinatorial optimization problem with four types of objectives, which are prioritized in the following order: (1) minimize the total number of buses required; (2) minimize the total travel time spent by all pupils; (3) minimize the total bus travel time; and (4) balance the loads and travel times among all buses. For an overview of the SBRP, we refer the reader to Park and Kim (2010).

In the SBRP, it is obvious that the demand at each pick-up point must be integer. When the vertex demand is allowed to be a real number, the resulting problem is called the *cumulative vehicle routing problem* (CumVRP) (Kara, Kara, & Yetiş, 2008; Ngueveu, Prins, & Wolfler Calvo, 2010; Ribeiro & Laporte, 2012). The CumVRP is the same as the classical *capacitated vehicle routing problem* (CVRP) (Toth & Vigo, 2002) except that the cost of traversing an edge is the product of length and flow of the edge. It has been shown that the MTRP is a special case of the delivery formulation of the CumVRP; we refer the reader to Kara et al. (2008) for details of the proof. The CumVRP can be further regarded as a special case of the weighted vehicle routing problem (WVRP) proposed by Zhang, Tang, Pan, and Yuan (2010). The total cost to be minimized in the WVRP consists of three components: (1) the fixed cost of dispatching a vehicle; (2) the cost per unit travel distance; and (3) the constant surcharge per unit weight per unit distance. Later, the WVRP was extended to include multiple depots by Zhang, Tang, and Fung (2011).

When the surcharge per unit weight per unit distance is a function of the vehicle weight, the WVRP is generalized to the vehicle routing problem with toll-by-weight scheme (VRPTBW) (Shen, Qin, & Lim, 2009; Zhang, Qin, Zhu, & Lim, 2012). To date, over twenty-five Chinese provinces have implemented the toll-by-weight schemes, all of which are monotonically increasing functions of the vehicle weight. Denoting by the decision variable $w_{i,j}$ the weight of the vehicle traversing edge (i, j) and assuming the surcharge per unit distance is calculated based on a toll function $f(w_{i,j})$, the objective of the VRPTBW is to minimize $\sum_{i\in V}\sum_{j\in V} d_{i,j}f(w_{i,j})$. Consider the case where the toll function has the following form:

$$f(w_{i,j}) = \begin{cases} 0 & \text{if } w_{i,j} = 0 \\ \alpha w_{i,j} + \beta & \text{if } w_{i,j} > 0 \end{cases}$$

where α and β are non-negative parameters. If $\alpha = 0$ and $\beta = 1$, then VRPTBW reduces to the classical CVRP problem. If $\alpha = 1$ and $\beta = 0$, then VRPTBW reduces to the CumVRP. If $\alpha > 0$ and $\beta = 0$, then $\sum_{i\in V}\sum_{j\in V} d_{i,j}f(w_{i,j})$ can be written as $\sum_{i\in V}\sum_{j\in V} \alpha d_{i,j}w_{i,j}$, which is actually the third cost component of the WVRP (Zhang et al., 2010).

The workover rig routing problem (WRRP) introduced by Aloise et al. (2006) is another variant of the MTRP. In the WRRP, a set of onshore oil wells needs maintenance service from a fleet of heterogeneous workover rigs. For each well, its production loss equals the product of the production loss rate and the time at which its required service is completed. The objective of this problem is to find a route for each workover rig such that the total production loss of the wells over a finite horizon is minimized. In recent years, the WRRP has also been studied by several other researchers, such as Pacheco, Ribeiro, and Mauri (2010), Ribeiro, Laporte, and Mauri (2012), and Ribeiro, Desaulniers, and Desrosiers (2012).

After reviewing prior studies with regard to the MTRP and its variants, we find that almost all relative articles except Ribeiro, Desaulniers, et al. (2012) proposed near-optimal algorithms, such as approximation algorithm (Fakcharoenphol et al., 2007), heuristics (Bennett & Gazis, 1972; Li & Fu, 2002), scatter search algorithms (Zhang et al., 2010, 2011), simulated annealing algorithm (Shen et al., 2009) and variable neighborhood search (Aloise et al., 2006). In Ribeiro, Desaulniers, et al. (2012), the authors proposed a branch-and-price-and-cut algorithm for the WRRP, where their column generation pricing subproblem was solved by a mono-directional label-setting algorithm.

In this paper, we provide an exact branch-and-price-and-cut algorithm for the MTRPD. The pricing subproblem of the MTRPD is called the *resource-constrained elementary shortest path problem with cumulative costs*. Although label-setting algorithms have been successfully applied to similar pricing subproblems in several previous articles, e.g., Ribeiro, Desaulniers, et al. (2012) and Ioachim et al. (1998), our work is the first attempt to develop bounded bidirectional label-setting algorithm for solving it. To the best of our knowledge, branch-and-price-and-cut method is the most successful exact algorithm for the vehicle routing models (Baldacci, Christofides, & Mingozzi, 2008; Fukasawa et al., 2006), such as the vehicle routing problem with time windows (VRPTW) (Desaulniers, Lessard, & Hadjar, 2008; Jepsen, Petersen, Spoorendonk, & Pisinger, 2008), the split delivery VRPTW (Archetti, Bouchard, & Desaulniers, 2011; Desaulniers, 2010), the capacitated location-routing problem (Contardo, Cordeau, & Gendron, 2011), the heterogeneous fleet vehicle routing problem (Pessoa, Uchoa, & de Aragão, 2009) and the pickup and delivery problem with time windows (Ropke & Cordeau, 2009).

The remainder of the paper is structured as follows. Section 2 presents an arc-flow formulation and a set-covering formulation for the MTRPD. This is followed in Section 3 with a description of column generation, consisting of the pricing subproblem, the label-setting algorithm for solving the pricing subproblem and three acceleration strategies. Subsequently, we present other main components of the branch-and-price-and-cut algorithm in Section 4. Our experimental results are given in Section 5, and we conclude our paper in Section 6 with some closing remarks.

2. Mathematical formulation

The arc-flow formulation of the MTRPD uses two types of decision variables: a binary decision variable $x_{i,j,k}$ that equals 1 if vehicle k directly travels from vertex i to vertex j, and 0 otherwise; and a non-negative variable $y_{i,k}$ that represents the time at which vehicle k arrives at vertex i. We denote by $V^{+}(i) = \{j \in V|(i, j) \in E\}$ and $V^{-}(i) = \{j \in V|(j, i) \in E\}$ the immediate successors and predecessors of vertex i in G. Letting F be the set of K vehicles and M be a sufficiently large positive number, the arc-flow formulation is given as: