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Discrete Optimization

A modified variable neighborhood search for the discrete ordered median problem

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ABSTRACT

This paper presents a modified Variable Neighborhood Search (VNS) heuristic algorithm for solving the Discrete Ordered Median Problem (DOMP). This heuristic is based on new neighborhoods' structures that allow an efficient encoding of the solutions of the DOMP avoiding sorting in the evaluation of the objective function at each considered solution. The algorithm is based on a data structure, computed in preprocessing, that organizes the minimal necessary information to update and evaluate solutions in linear time without sorting. In order to investigate the performance, the new algorithm is compared with other heuristic algorithms previously available in the literature for solving DOMP. We report on some computational experiments based on the well-known *N-median* instances of the ORLIB with up to 900 nodes. The obtained results are comparable or superior to existing algorithms in the literature, both in running times and number of best solutions found.

1. Introduction

Location analysis is a very active topic within the Operations Research community. It has given rise to a number of nowadays standard optimization problems some of which are in the core of modern mathematical programming. One of its more important branches is Discrete Location. Witnesses of its importance are a number of survey articles and textbooks that collect a large number of references on methodological results and applications, see e.g. Daskin (1995), Drezner and Hamacher (2002), Mirchandani and Francis (1990), Nickel and Puerto (2005) and references therein. Roughly speaking, Discrete Location problems typically involve a finite set of *sites* at which facilities can be located, and a finite set of *clients*, whose demands have to be satisfied from the facilities.

An important aspect of a location model is the right choice of the objective function and in most classical location models the objective function is the main differentiator. Therefore, a great variety of objective functions has been considered.

Discrete Ordered Median Problem was introduced to provide a unifying way to model many location models see e.g. Nickel (2001), Boland, Domínguez-Marín, Nickel, and Puerto (2006) and Nickel and Puerto (2005). It has been recognized as a powerful tool from a modeling point of view because it generalizes the most popular objective functions in the literature of location analysis and it also allows to distinguish the different roles played by the different parties in a supply chain network. The correct identification of the different roles played by the agents participating in logistics models has led to describe and analyze new types of distribution patterns, namely customer-oriented, supplier-oriented or third party logistics provider-oriented, see (Kalcsics, Nickel, Puerto, & Rodríguez-Chía, 2010a, 2010b; Puerto, Ramos, & Rodríguez-Chía, 2011). The objective function of DOMP applies a penalty to the cost of supplying a client which is dependent on the *position* of that cost relative to the costs of supplying the remaining clients. Therefore, it increases the flexibility in the modeling phase through rank dependent compensation factors which allow to model which party is the driving force in a supply chain.

In the last years, a number of algorithms have been developed to attack the resolution of DOMP (see Boland et al., 2006; Kalcsics et al., 2010a, 2010b; Marín, Nickel, Puerto, & Velten, 2009; Nickel, 2001; Nickel & Puerto, 1999 & Rodríguez-Chía et al., Rodríguez-Chía, Nickel, Puerto, & Fernández, 2000). The first exact method was a branch and bound (B& B) algorithm presented in Boland et al. (2006). Later, a more specialized formulation was introduced in Marín et al. (2009) giving rise to a more efficient branch and cut (B&Cut) algorithm. Finally, in Marín, Nickel, and Velten (2010) the authors develop a new formulation that has allowed to solve larger size instances to optimality. The reader is referred to Marín et al. (2010) for a comprehensive literature review on exact methods for DOMP. More recently, the capacitated version of these

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problems has been also considered in Kalcsics et al., 2010a, 2010b where first attempts to solve capacitated versions of DOMP have been developed. However, none of these approaches leads to satisfactory results concerning the solution times of even medium size instances. In spite of that, the literature on heuristic algorithms for this family of problems is rather reduced. Domínguez-Marín, Nickel, Hansen, and Mladenović (2005) present two heuristic approaches, a Variable Neighborhood Search (VNS) and a genetic algorithm, for solving DOMP, whereas Stanimirovic, Kratica, and Dugosija (2007) proposes two Evolutionary Programs (with two different encodings: binary in HGA1 and integer in HGA2) based on new encodings of the solution for better evaluation of the objective function that improve the heuristic algorithms in Domínguez-Marín et al. (2005). In both papers, the authors use ORLIB N-median instances with up to 900 nodes for testing their results.

The goal of this paper is to develop a modified VNS heuristic for DOMP which takes some advantage of the new available knowledge on the structure of this problem. Specifically, we use refined neighborhoods' structures that favor faster improvement of the objective function in the local search phase. Moreover, we apply an efficient encoding of solutions avoiding sorting in each objective function evaluation. This specific encoding allows to obtain a faster implementation than the one in Domínguez-Marín et al. (2005) for the VNS paradigm to this family of problems. In addition, it provides results that compare with the best heuristic algorithms known so far for the DOMP (Stanimirovic et al., 2007). To this end, the paper is organized as follows. Section 2 is devoted to recall the DOMP. Section 3 describes our modified VNS algorithm for the DOMP. There, we describe our neighborhoods' structures and the different elements that allow a better encoding of solutions avoiding sorting in each evaluation of the objective function. The presentation of the algorithm is modular. Thus, we present the different functions that are used in the algorithm, namely, *Initial Solution*, *Variable Neighborhood Descent*, *Shaking* and finally the actual algorithm *Modified Variable Neighborhood Search*. In Section 4, we report our computational results based on 8 problem types, previously considered in the literature, and on data taken from the benchmark instances of the ORLIB N-median library by Beasley (1990). The paper ends with some conclusions on the proposed algorithm and on the comparisons with previously available heuristics for the considered problem.

2. The discrete ordered median problem

In order to introduce the *Discrete Ordered Median Problem* (DOMP) formally, we define a set V of M discrete locations. These locations represent clients as well as potential plant locations.

Moreover, let $C = [c_{ij}]$ $(i, j = 1, \dots, M)$ be a non–negative $M \times M$ cost matrix, whereas c_{ij} denotes the cost of satisfying the total demand of client i from a plant at location j. Thereby, we assume that $c_{ii} = 0$ $(\forall\ i = 1, \dots, M)$. This property of C is called *free self–service* (FSS). For the sake of readability, we denote by G the number of different values assumed by the elements of matrix C. Moreover, we shall refer to the sorted values of C by $c_{(j)}$, $j = 1, \dots, G$.

Let N with $1 \leqslant N \leqslant M - 1$ be the number of new plants which have to be located at the candidate sites. Then, the costs for satisfying the demand of the respective clients, given a feasible solution $X \subset V$ with $|X| = N$, can be represented by the following vector

$$c(X) := \ (c_1(X), \dots, c_M(X)) \quad \text{with } c_i(X) = \min_{j \in X}\{c_{ij}\}\ \forall\ i \in V.$$

However, due to the desired flexibility, $c(X)$ cannot directly be used to define the objective function of the DOMP. Instead, consider a permutation σ_X on $\{1,\dots,M\}$ for which the inequalities

$$c_{\sigma_X(1)}(X) \leqslant c_{\sigma_X(2)}(X) \leqslant \cdots \leqslant c_{\sigma_X(M)}(X)$$

hold. Using this permutation we define the *sorted cost vector* $c_{\leqslant}(X)$ corresponding to a feasible solution X as follows:

$$c_{\leqslant}(X) := (c_{\sigma_X(1)}(X), \dots, c_{\sigma_X(M)}(X))$$

or for short

$$c_{\leqslant}(X) := (c_{(1)}(X), \dots, c_{(M)}(X)).$$

Furthermore, let $\lambda = (\lambda_1, \dots, \lambda_M)$ be an M-dimensional vector, with $\lambda_i \geqslant 0$ $(\forall\ i = 1, \dots, M)$ representing a weight on the ith lowest component of the cost vector $c(X)$. Using the notation explained above the DOMP is defined as:

$$\min_{\substack{X \subset V \\ |X|=N}} f_\lambda(X) = \sum_{i=1}^{M} c_{(i)}(X) \cdot \lambda_i. \tag{1}$$

The function $f_\lambda(X)$ is called *ordered median function*. An example illustrating the structure of the DOMP and the calculation of the ordered median function is given below.

Example 2.1. Let $V = \{1, \dots, 5\}$ and assume that $N = 2$ plants have to be located. Moreover, let the cost matrix C be as follows:

$$C = \begin{pmatrix} 0 & 4 & 5 & 3 & 3 \\ 1 & 0 & 6 & 2 & 2 \\ 7 & 3 & 0 & 3 & 1 \\ 7 & 3 & 5 & 0 & 5 \\ 1 & 3 & 2 & 3 & 0 \end{pmatrix}.$$

Clearly $G = 8$ and $c_{(1)} = 0$, $c_{(2)} = 1$, $c_{(3)} = 2$, $c_{(4)} = 3$, $c_{(5)} = 4$, $c_{(6)} = 5$, $c_{(7)} = 6$, $c_{(8)} = 7$.

With $\lambda = (0, 0, 1, 1, 0)$, an optimal solution of this problem instance is $X = \{1, 4\}$. Therefore, the demand of locations 1, 2 and 5 are satisfied by plant 1 whereas the demand of the remaining locations are satisfied by plant 4. Hence, $c(X) = (0, 1, 3, 0, 1)$, $c_{\leqslant}(X) = (0, 0, 1, 1, 3)$ and

$$f_\lambda(X) = 0 \cdot 0 + 0 \cdot 0 + 1 \cdot 1 + 1 \cdot 1 + 0 \cdot 3 = 2.$$

Note that by using appropriate values for λ, nearly all classical discrete facility location problems can be modeled by the above definition. In addition, a wide range of new and interesting problems can be derived. Some of these modeling possibilities are given in Table 1. For a more extensive list the interested reader is referred to Domínguez-Marín (2003) and Nickel and Puerto (2005).

Since the DOMP contains, as a special instance, the discrete N-median problem which is $\mathcal{NP}$-hard (see Kariv & Hakimi, 1979) the DOMP is also $\mathcal{NP}$-hard. In spite of that, as mentioned in the Introduction, different integer linear programming formulations have also been proposed for DOMP which can solve to optimality medium size instances up to 100 nodes (Marín et al., 2009). Nevertheless, for larger sizes the exact approaches do not perform well. In the following section we propose a modified VNS heuristic for DOMP.

Table 1
Modeling possibilities.

λ	$f_\lambda(X)$	Meaning
$(1, \dots, 1)$	$\sum_{i=1}^{M} c_i(X)$	N-Median
$(0, \dots, 0, 1)$	$\max_{1 \leqslant i \leqslant M} c_i(X)$	N-Center
$(\alpha, \dots, \alpha, 1)$ $\alpha \in [0, 1]$	$\alpha \cdot \sum_{i=1}^{M} c_i(X) + (1-\alpha) \cdot \max_{1 \leqslant i \leqslant M} c_i(X)$	α-Centdian
$(0, \dots, 0, \underbrace{1, \dots, 1}_{k})$	$\sum_{i=M-k+1}^{M} c_{(i)}(X)$	k-Centrum