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Cold vs. hot standby mission operation cost minimization for 1-out-of-*N* systems

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ABSTRACT

It is well recognized that using the hot standby redundancy provides fast restoration in the case of failures. However the redundant elements are exposed to working stresses before they are used, which reduces the overall system reliability. Moreover, the cost of maintaining the hot redundant elements in the operational state is usually much greater than the cost of keeping them in the cold standby mode. Therefore, there exists a tradeoff between the cost of losses associated with the restoration delays and the operation cost of standby elements. Such a trade-off can be obtained by designing both hot and cold redundancy types into the same system. Thus a new optimization problem arises for the standby system design. The problem, referred to in this work as optimal standby element distributing and sequencing problem (SE-DSP) is to distribute a fixed set of elements between cold and hot standby groups and select the element initiation sequence so as to minimize the expected mission operation cost of the system while providing a desired level of system reliability. This paper first formulates and solves the SE-DSP problem for 1-out-of-*N*: G heterogeneous non-repairable standby systems. A numerical method is proposed for evaluating the system reliability and expected mission cost simultaneously. This method is based on discrete approximation of time-to-failure distributions of the system elements. A genetic algorithm is used as an optimization tool for solving the formulated optimization problem. Examples are given to illustrate the considered problem and the proposed solution methodology.

1. Introduction

Standby redundancy is a technique in which one or multiple modules are on-line and operational with one or more modules serving as standby units. When an on-line unit becomes faulty, it is removed from operation and replaced with a standby unit. Using the standby redundancy technique to improve the reliability of a system or subsystem has become a well-known principle in the system reliability engineering field (Johnson, 1989). Examples abound in the real-world systems, such as airplanes with multi-engine systems, cables in a bridge, data processing systems with multiple video displays, and communication systems with multiple transmitters (Amari & Pham, 2010; Amari, Zuo, & Dill, 2008).

There are two traditional types of standby redundancy: *hot* and *cold*. In hot standby redundancy, elements that are in the standby mode operate in synchrony with the on-line primary unit and are ready to take over at any time; while in cold standby redundancy, elements in the standby mode are unpowered and thus do not operate until needed to replace a faulty on-line unit (Johnson, 1989). Hot standby redundancy can provide fast restoration in the case of failures. However because the hot standby elements are fully exposed to working stresses, they can fail even before they are used, which reduces the overall system reliability. In contrast, the cold standby elements require long restoration delays when they are needed to operate as a substitute for a failed online element. However they are shielded from the working stresses associated with system operation; the likelihood of failure in the cold standby mode is very low and can be assumed to be zero. Therefore, using cold standby redundancy can usually provide higher system reliability than using hot standby redundancy.

From the mission cost point of view, the cost of losses associated with the restoration delays in a hot standby redundancy system is much lower than that in a cold standby redundancy system. However, maintaining the redundant elements in the hot standby mode is usually much more costly than keeping them in the cold standby mode as working elements consume more energy and materials. Therefore, there exists a tradeoff between the restoration cost and the operation cost of standby elements. A compromise can be obtained by designing both types of redundancies into the same system. Consider a 1-out-of-*N*: G standby system

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Nomenclature

N	number of elements in the system
H	number of HS elements
$s(i)$	index of the element initiated after $i-1$ failures
T_j	*r.v.* representing the time-to-failure of element j
η_j, β_j	scale and shape parameters of element j with Weibull failure distribution
$p_j(i)$	probability that element j fails in time interval i after its initiation
w_j	cost (per time unit) of keeping element j in hot standby (or operation) mode
W_j	cost (per time unit) of keeping element j in cold standby mode
V_j	startup cost of cold standby element j
v_j	startup cost of hot standby element j
τ	mission time
R	system reliability
C	expected mission cost
X_i	random cumulative working time of elements $s(1)$, $s(2)$, ..., $s(i)$: $X_i = \min\left\{\tau, \sum_{j=1}^{i} T_{s(j)}\right\})$
$Q_j(i)$	$\Pr\{X_j = \Delta i\}$
m	number of considered time intervals during the mission
Δ	duration of each time interval
$\theta_j(t)$	total cost of using HS element j failed or switched off at time t
$\Theta_j(t_0, t)$	total cost of using CS element j initiated at time t_0 and failed or switched off at time t

Acronyms

cdf	cumulative distribution function
pdf	probability density function
pmf	probability mass function
r.v.	random variable
GA	genetic algorithm
HS	hot standby
CS	cold standby
SE-DSP	standby element distributing and sequencing problem

designed with a fixed set of elements. The distribution of the system elements between hot and cold standby modes and the order in which the standby elements are initiated heavily affect the system reliability as well as mission cost (associated with energy consumption, standby and operation maintenance, startup expenses, costs of restoration delays, etc.). Thus a new optimization problem arises, which distributes the elements between cold and hot standby groups and selects the element initiation sequence with the objective to minimize the expected mission cost of the system while providing a desired reliability level. We refer to such problem as optimal standby element distributing and sequencing problem (SE-DSP). In this paper, we first formulate and solve the SE-DSP problem using a combination of a numerical method based on a discrete approximation of time-to-failure distributions of the system elements and a genetic algorithm. The numerical method is used for evaluating the system reliability and expected mission cost; the genetic algorithm is used as an optimization tool for solving the formulated optimization problem.

Considerable research efforts have been expended in formulating and solving optimization problems such as redundancy allocation and reliability allocation problems for standby systems (Gen & Yun, 2006; Kuo, Prasad, Tillman, & Hawang, 2001; Kuo & Wan, 2007). For example, exact optimization methods like dynamic programming and integer programming were proposed for solving the redundancy allocation problem (RAP) of 1-out-of-N: G *homogeneous* hot-standby series-parallel systems, where one type of elements can be substituted only by the same type of elements to achieve fault tolerance (Fyffe, Hines, & Lee, 1968; Misra & Sharma, 1991). Meta-heuristic optimization methods like genetic algorithm and Tabu search were proposed to solve the RAP of 1-out-of-N: G or K-out-of-N: G *heterogeneous* hot-standby series-parallel systems, where one type of elements can be substituted with a different type of functionally-equivalent elements to achieve fault tolerance (Chen & You, 2005; Chia & Smith, 2004; Coit & Smith, 1996; Onishi, Kimura, James, & Nakagawa, 2007). In Coit (2001), an integer programming-based method was proposed to solve the redundancy optimization problem for 1-out-of-N: G cold-standby series-parallel systems. Furthermore, in Coit and Liu (2000), Coit (2003) the integer programming-based method was applied to solve the RAP for 1-out-of-N: G or K-out-of-N: G heterogeneous series-parallel systems with combined redundancies where each subsystem involves either hot or cold standby redundancy. It is worth noting that there also exist works considering both standby redundancy and active redundancy (Chambari, Rahmati, Najafi, & Karimi, 2012; Zhao & Liu, 2004). In the active redundancy technique, redundant elements are put in parallel to elements of the system for successful operation without using switching mechanisms; such technique is used when the replacement of elements during system operation is impossible (Bueno & Carmo, 2007; Zhao, Chan, & Ng, 2012). Particularly in Chambari et al. (2012), the RAP was solved in a bi-objective reliability-cost formulation using multi-objective version of the genetic algorithm for systems with choices of active or cold-standby redundancies for individual subsystems. In Zhao and Liu (2004) a hybrid intelligent algorithm based on genetic algorithm, neural networks, and fuzzy theory was proposed to solve the RAP for systems with uncertain element lifetimes where redundant elements are arranged in either active or cold-standby. To the best of our knowledge, none of the above-mentioned works have considered the proposed optimization problem that minimizes the expected mission operation cost for systems or subsystems with mixed hot and cold redundancy types.

The remainder of the paper is organized as follows: Section 2 describes the system we considered and assumptions used in this work. Section 3 presents the numerical method for evaluating the system reliability and expected mission cost of a 1-out-of-N: G heterogeneous non-repairable standby system with a mix of hot and cold standby elements. Section 4 presents numerical examples and results for illustrating the proposed numerical method. Sections 5 and 6 present the formulation and examples for the considered optimization problem. Lastly, Section 7 gives conclusion as well as directions for future work.

2. The model

The system consists of N elements of which H elements are in the hot standby (HS) mode and $N-H$ elements are in the cold standby (CS) mode. One working element can successfully accomplish the system mission, *i.e.* the system is of 1-out-of-N: G type. All the HS elements are initialized at the beginning of the mission. When a failed element is replaced with an HS element the