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Coordinating lead times and safety stocks under autocorrelated demand

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ABSTRACT

We consider a supply chain in which orders and lead times are linked endogenously, as opposed to assuming lead times are exogenous. This assumption is relevant when a retailer's orders are produced by a supplier with finite capacity and replenished when the order is completed. The retailer faces demands that are correlated over time – either positively or negatively – which may, for example, be induced by a pricing or promotion policy. The auto-correlation in demand affects the order stream placed by the retailer onto the supplier, and this in turn influences the resulting lead times seen by the retailer. Since these lead times also determine the retailer's orders and its safety stocks (which the retailer must set to cover lead time demand), there is a mutual dependency between orders and lead times. The inclusion of endogenous lead times and autocorrelated demand represents a better fit with real-life situations. However, it poses some additional methodological issues, compared to assuming exogenous lead times or stationary demand processes that are independent over time. By means of a Markov chain analysis and matrix analytic methods, we develop a procedure to determine the distribution of lead times and inventories, that takes into account the correlation between orders and lead times. Our analysis shows that negative autocorrelation in demand, although more erratic, improves both lead time and inventory performance relative to IID demand. Positive correlation makes matters worse than IID demand. Due to the endogeneity of lead times, these effects are much more pronounced and substantial error may be incurred if this endogeneity is ignored.

1. Introduction

In this paper we study the issue of coordinating the retailer's inventory decisions and the supplier's lead times. It is commonly known that supplier lead times have a direct impact on the retailer's safety stocks: longer and more variable lead times require higher safety stocks. But in a make-to-order setting there is also an impact in the opposite direction: the lead times vary according to the order stream of the retailer and its variability. Complicating matters is the assumption that the retailer may be facing orders that are correlated over time. The degree of autocorrelation (and whether it is positive or negative) greatly impacts the level of fluctuations in the order stream, influencing in turn the lead time distribution. In addition we will show that, due to this autocorrelation, the order stream becomes dependent upon the lead time distribution. The objective of this paper is to study the interplay between the correlation in demand, the retailer's order policy (and its safety stocks) and the supplier's lead times. This interplay has, to the best of our knowledge, not been dealt with in the literature before. The resulting production/inventory system poses some challenging methodological issues.

Coordinating lead times and safety stocks is imperative in a supply chain where the supplier produces the retailer's orders on a make-to-order basis. Several reasons may motivate a make-to-order approach, ranging from a limited shelf life to frequent upgrades or customer's specific packaging requirements. In such an environment the supplier may opt to not hold inventory, but the retailer does hold safety stocks to satisfy immediate consumer demand. We have encountered several examples where a make-to-order policy is employed for customized products, and where the insights obtained in this paper can be applied. For instance, an industrial bakery, producing authentic specialities in the biscuit and cake world, employs a make-to-order policy for a major retailer due to specific packaging requirements with the retailer's label on the product, sometimes combined with a specific, temporary promotion. As the products have a limited shelf life and the retailer's orders fluctuate every period, a make-to-stock policy is not suitable for these products. Another example is a supplier of

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feminine-care and baby-care products (diapers, baby wipes, tampons, etc.) who manufactures retailer brands. In their quest to compete with A-brands, they rely heavy on promotions. Due to the high fluctuations in demand, in combination with the retailer-specific requirements, the supplier does not keep any stock, instead he produces to order. The retailer however holds the product in inventory to ensure immediate availability to the consumer.

The only abstraction we make from these practical settings is the fact that we apply our methodology to a single item; however our insights can be generalized to a multi-item setting, where a safety stock is held per item. Our model is also capable of representing a firm that replenishes its finished goods inventories from its own production facilities. This firm must plan releases into the production system in such a manner as to maintain safety stocks at its inventory locations facing customers.

In such a make-to-order setting the nature of the order stream (variability in inter-arrival time and order sizes) affects the sojourn times within the supplier's queue, and thus the lead times observed by the retailer. By modeling a two-echelon (retailer-manufacturer) supply chain as a production/inventory system, we treat lead times as *endogenous* variables; this means that we do not merely assume the replenishment lead time to be a fixed or random *exogenous* variable. Instead we include the impact of a replenishment decision on the production lead times and use these lead times in our inventory model. We use an iterative procedure to cope with this interaction effect.

The inclusion of autocorrelation (or time-correlation) in demand, as opposed to assuming IID demand, is valid in many high-tech and consumer goods industries (see e.g. Dong & Lee, 2003). In these industries, consumers are typically highly sensitive to marketing actions. We analyzed a large number of consumer demand patterns (weekly POS data) for consumer packaged non-food products, both branded products and private label products. For the regular 'turn' business, positively autocorrelated demand patterns seem to dominate. This is confirmed by Erkip, Hausman, and Nahmias (1990) and Disney, Farasyn, Lambrecht, Towill, and Van de Velde (2006) who also find that positively correlated consumer demand was most commonly observed. However, in the presence of recurring weekly promotions, a retailer may observe negative autocorrelation as well; this is due to consumers stockpiling during the promotion period and cannibalising demand before (and after) the promotion. In the marketing literature, this is referred to as pre- and post-promotion dips (Macé & Neslin, 2004). This promotion strategy may create negative period-to-period correlation in demand.

We show that correlation in demand has an important impact on the performance of the supply chain in terms of safety stocks and lead times. The inclusion of autocorrelation in demand illustrates that price control mechanisms can be used to manage supply chains, reinforcing once more the importance of coordinating marketing and operations decisions along the chain. Note that in this paper we focus on the impact of autocorrelation, rather than on the overall variability in demand, which can also be influenced by price promotions. We refer to Raju (1992) who relates the promotional activity in a product category to its variability in sales and Boute, Lambrecht, and Van Houdt (2007) who study the operational impact of demand variability on lead times and safety stocks.

The remainder of this paper is organized as follows. The next section presents an overview of the related literature. Section 3 describes our research model and derives expressions for the orders generated by the retailer. Section 4 develops an iterative procedure to determine the endogenous supply lead times and Section 5 is devoted to the analysis of the safety stocks in the combined production/inventory system. Section 6 provides a numerical experiment and Section 7 concludes.

2. Literature review

This paper studies the interplay between autocorrelation in demand, the retailer's inventory policy (facing the autocorrelated demand), and the supplier's lead times (producing the retailer's orders). In the literature inventory models are discussed with either autocorrelated demand, but assuming exogenous lead times; or production/inventory models with endogenous lead times, assuming IID demand. In the following we will review the literature on both these streams of research. We will also discuss the literature on our methodology used.

Several papers discuss supply chains with autocorrelated demand and constant (exogenous) lead times. Fotopoulos, Wang, and Rao (1988) provide an upper bound for the safety stock when daily demands are autocorrelated and lead times follow an arbitrary distribution. Erkip et al. (1990) derive optimal stocking levels as a function of the autocorrelation coefficient. Dong and Lee (2003) develop a lower bound for the optimal stocking levels in serial multi-echelon systems under time-correlated demand. Kahn (1987) and Lee, Padmanabhan, and Whang (1997) demonstrate the existence of variance amplification upstream in the chain (aka the bullwhip effect) when the retailer follows a base-stock policy and demand is positively correlated. Zhang (2004b) studies the role of forecasting for AR (1) demands and concludes that the minimum Mean Squared Error (MSE) forecasting method minimizes the variance of the forecasting error among all linear forecasting methods, and therefore leads to the lowest inventory costs. Alwan, Liu, and Yao (2003) employ this optimal MSE forecasting scheme and determine the underlying time-series model of the resulting order process. They show that when consumer demand is negatively correlated (with AR demand), the variability in orders is dampened with respect to the observed demand. This result is of great importance for our paper.

Negative correlation may occur, for instance, due to consumers stockpiling during the promotion period and deceleration before and after the promotion. Stockpiling is the propensity of consumers to increase their inventories above normal levels either by purchasing the category earlier, or by purchasing greater-than-normal quantities (Neslin, Henderson, & Quelch, 1985). Deceleration is the willingness of consumers to deplete their inventories below normal levels by 'holding out' for an anticipated promotion (Mela, Jedidi, & Bowman, 1998). The impact of promotions on consumer demand behavior is extensively discussed in the marketing literature as they may influence profitability (Blattberg & Neslin, 1993; Hendel & Nevo, 2006). Macé and Neslin (2004) empirically studied the relationships between pre- and post-promotion dips in weekly store data, and find that these dips are stronger for high-priced, frequently promoted, mature, high-market-share products.

The interaction between inventory policies and lead times is generally studied in production/inventory systems with endogenous lead times. Graves (1988) provides an excellent review and critique of the research literature on safety stocks for manufacturing systems, and proposes a model to include consideration of the flexibility of the production stage in planning safety stocks. Base-stock controlled production/inventory systems in continuous time with exponential (single unit) demand processes have been studied widely, among many others, by Gavish and Graves (1980, 1981), Song and Zipkin (1996), Sox, Thomas, and McClain (1997) and Jemaï and Karaesmen (2005). Ettl, Feigin, Lin, and Yao (2000) and Liu, Liu, and Yao (2004) model a supply network with multiple storage locations by means of an inventory-queue model assuming Poisson demand. Boute et al. (2007) propose a solution method for production/inventory systems in discrete time with a random IID integer consumer demand. However, none of these papers consider autocorrelation in demand. The interaction between order release