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# Reserve stock models: Deterioration and preventive replenishment ☆

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ABSTRACT

Reserve stocks are needed in a wide spectrum of industries from strategic oil reserves to tactical (machine buffer) reserves in manufacturing. One important aspect under-looked in research is the effect of deterioration, where a reserve stock, held for a long time, may be depleted gradually due to factors such as spoilage, evaporation, and leakage. We consider the common framework of a reserve stock that is utilized only when a supply interruption occurs. Supply outage occurs randomly and infrequently, and its duration is random. During the down time the reserve is depleted by demand, diverted from its main supply. We develop optimal stocking policies, for a reserve stock which deteriorates exponentially. These policies balance typical economic costs of ordering, holding, and shortage, as well as additional costs of deterioration and preventive measures. Our main results are showing that (i) deterioration significantly increases cost (up to 5%) and (ii) a preventive replenishment policy, with periodic restocking, can offset some of these additional costs. One side contribution is refining a classical reserve stock model (Hansmann, 1962).



## 1. Introduction and literature review

An implicit assumption in traditional inventory control models is the continuity of supply, where the potential for supply interruption is not considered. Supply interruptions can occur due to internal random incidents such as machine breakdown, or due to external random events such as worker strikes or political crises. For example, the internal operations of a factory can be disrupted if an upstream operation fails in a tandem factory layout; thus, starving the downstream machine. External random occurrences of natural disasters, wars, or strikes could also disrupt the incoming flow of material and render simple inventory control techniques inapt. A natural remedy to such situations is to resort to multi-sourcing solutions or simply carry a strategic/operational reserve stock that hedges against such random occurrences of internal and external disruptions. However, another complicating factor that is usually ignored in research is the deterioration of reserve stock. Deterioration, which could take place to any reserve stock (strategic or operational) is more pronounced in situations where the storage amounts are large and storage periods are long. Frequent monitoring of such reserves is necessary to ensure that the quality and the quantity of these reserves are maintained during long storage periods and are available when needed.

Strategic oil reserves are important examples of reserve stock where deterioration can be an important factor. For example, the US government carries a strategic petroleum reserve of about 725 million barrels of oil (about 75 days of import protection) in underground salt caverns at five sites along the Gulf of Mexico. These strategic reserves are kept for long periods and used only in case of emergency disruptions. Europe, Asia, and India are also following US footsteps in developing their own strategic storage programs (Thomson, 2009). Giles, Joachim Koenig, Neihof, Shay, and Woodward (1991) discussed the deterioration of such reserves and find that refined products tend to have less storage stability than crude oils. This finding is corroborated by data from the Canadian Ministry of Agriculture showing that evaporation losses from gasoline storage tanks is up to 3.2% per summer month (FSHR, 2005).

In manufacturing, large reserves are kept, and deterioration may occur. For example, according to the US Census Bureau, the total value of manufacturing inventories was around $610 billion in December 2011. Among these, there are $240 billion dollars in nondurable goods inventory (such as food, beverages, petroleum and chemical products), where deterioration may occur at a significant level.

In this paper, we combine both factors of random disruption and reserve stock deterioration, to build new models, which are more sensible in such uncertain environments. Next we present a concise review of the literature.

Our proposed research relates to three streams of research on inventory models with (i) supply disruption, (ii) deterioration,

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and (iii) corrective and preventive actions. In the following, we briefly review the research in these three streams.

First, on supply disruption, a seminal work is that by Hansmann (1962). He determined the optimal stock level by minimizing a cost function composed of holding and shortage costs. Our models are based on Hansmann's work. However, we account for additional costs related to ordering, deterioration and preventive replenishment. More recent works on supply disruption include Parlar and Berkin (1991) who analyzed an inventory control problem with exponentially distributed on (supplier available) and off (supplier unavailable) periods and deterministic demand, as we assume in this paper. In addition, Parlar (1997) introduced a model with random demand and random lead-time assuming that supply may be disrupted.

Few papers model supply disruptions as they relate to strategic reserves. Oren and Hang Wan (1986) determined the optimal size, fill up, and drawdown rates for a strategic petroleum reserve under a variety of supply and demand conditions using numerical methods. They assumed random on/off supply and deterministic demand, as we do in this paper. However, our model is different. We consider the additional factor of reserve deterioration, which is not studied by Oren and Hang Wan (1986). Wu and Weil (2010) developed a reserve stock policy that minimizes the expected insecurity cost to the Chinese economy arising from uncertainty in the supply of imported oil using a dynamic programming model. Zhang and Li (2009) developed a multi-objective model for managing strategic reserves with the objectives of minimizing storage equipment investment, storage expenses material value loss, and shortage risk. Both papers by Zhang and Li (2009) and Wu and Weil (2010) also ignored deterioration, unlike our models in this paper.

In a recent work, Shen, Dessouky, and Ordonez (2011) developed a model for a manufacturer of deteriorating medical supplies used as a strategic reserve by the US government in the event of a public health emergency. This work is closely related to this paper. However, Shen et al. (2011) research is different in two aspects (i) they took the point of view of the manufacturer, while we take the point of view of the reserve keeper (e.g. the government) and (ii) they assumed that deterioration occurs after a fixed lifetime, while we assume a random lifetime.

Second, on deteriorating or perishable inventories, it has been early recognized that special control policies are needed for such systems (see Nahmias (1982) for background and literature review). Random lifetime perishables, such as the one we consider in this paper, include oil produce, meats, and many other food products. One main ingredient of the research in this direction is the probability distribution of lifetime. The exponential lifetime distribution, which we adopt, is the most popular in the literature. The first known work that considered deterioration is by Ghare and Schrader (1963) who adapted the classic economic order quantity model to items with exponentially distributed lifetime. Several papers extended the work of Ghare and Schrader (1963) to account for backordering and finite production capacity, e.g., Misra (1975), Shah (1977) and Mak (1982). Reviews on perishable inventory models are presented by Raafat (1991) and Goyal and Giri (2001). This literature dealt with perishable products within an inventory control setting under continuous demand. However, in this paper, we investigate the effect of deterioration on a reserve stock where demand occurs sporadically when supply is disrupted, which constitutes a novel contribution.

Finally, inventory models on corrective actions typically relate to repair. For example, Chakrababorty, Giri, and Chaudhuri (2008) considered production lot sizing with machines which are prone to failure. They differentiated between corrective repair, which is done if a breakdown occurs during a production run, and preventive repair, which is done on a regular basis to minimize the chance of failure. Similar corrective/preventive repair models are considered by many other authors, e.g., Abboud (1997, 2001), Giri and Dohi (2005), and Lin and Gong (2006). Lin and Gong (2006) also considered exponentially deteriorated items, similar to our case. However, their focus was on a production lot sizing rather than on a reserve stock, like our case. A recent work which is more related to this paper is by Lee and Wu (2006) who considered statistical process control (SPC) based replenishment. This is shown to be effective in reducing the bullwhip effect in a two-echelon supply chain. The SPC based replenishment is somewhat similar to our proposed preventive replenishment scheme as both policies are triggered by signals related to the supply process. (In our work, the signal is the reserve stock dropping below a certain level.)

The rest of this paper is organized as follows. In Section 2, we present a base model adapted from Hansmann (1962) but with an additional variable ordering cost, which we found to have an important effect on the optimal stock level and the total cost. Our core contribution is in Section 3 where we account for the effect of exponential deterioration on the reserve stock. In Section 4, and in an effort to mitigate the effect of deterioration on reserve stock, we assume that a periodic preventive replenishment policy is adopted and we analyze the cost implications. In Section 5, we present numerical results and managerial insights. We find that neglecting deterioration can increase costs up to 5%, and that preventive replenishment can offset a significant part of these additional costs. Finally, Section 6 concludes the paper and discusses future research.

## 2. Base model

Consider a reserve stock which is held to hedge against supply interruption. Supply availability time, i.e. "up time", $X$, is a random variable with mean $1/\lambda$. Likewise, supply interruption time, i.e. "down time", $Y$, is a random variable with mean $1/\mu$. We make the reasonable assumption that down time is short relative to up time, $Y \ll X$. A base stock policy is adopted; a reserve stock of level $S$ is kept at all times when supply is available. In the event of a supply interruption, this reserve stock is consumed at a known rate $D$ per unit time. In the event that demand during the down time exceeds supply $S$, excess demand is lost. At the end of the supply interruption the reserve stock is replenished instantaneously up to $S$. The assumption of instantaneous replenishment is not restrictive. We adopt it to simplify the presentation. Fig. 1 shows a typical profile of stock level over time. Table 1 presents the notation used in this section and the remainder of the paper.

The company's cost structure entails (i) a holding cost proportional to average inventory with a unit cost of $h$ \$/unit/unit time, (ii) a shortage cost proportional to shortage time with a unit cost $\pi$ \$/unit time, and (iii) a variable ordering cost proportional to the amount ordered at the end of supply interruption with a unit cost $c$ \$/unit time. The variable ordering cost must be accounted for because excess demand during down time is lost. Previous literature (e.g. Hansmann, 1962) ignores this fact and neglects accounting for the ordering cost.

Under the assumption that $Y \ll X$, the drop in inventory during down time can be ignored. It follows that the expected holding cost per unit time is

$$C_{hu}(S) = hS. \qquad (1)$$

We define a *cycle* as the time between two consecutive supply restorations. Then, with the assumption that $Y \ll X$, the expected cycle duration is approximately $1/\lambda$. The shortage cost depends on whether demand exceeds supply during down time. Therefore, the expected shortage cost per cycle is