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Decision Support

Outranking under uncertainty using scenarios

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ABSTRACT

This paper considers the use of scenarios to treat uncertain attribute evaluations in the outranking methods. The scenario-based approach allows the decision maker to think deterministically about the problem by attaching causal links to a small number of potential outcomes, instead of using probability distributions. The scenario approach can be expressed as a simplified version of the comprehensive but practically complex "distributive" outranking method of d'Avignon and Vincke. Using a scenario approach has distinct practical advantages, but also presents the inherent danger that meaningful information is ignored. The extent of this danger is assessed using a simulation experiment, where it is found to be of a magnitude that is non-trivial but still potentially acceptable for certain decision contexts.

1. Introduction

Many decisions must be made in conditions where the consequences of some actions are unknown because they depend on future events. This is sometimes termed "external uncertainty" (e.g. Stewart, 2005, chap. 11) because it relates to uncertainty about environmental conditions lying beyond the control of the decision maker. Particularly in the realm of strategic decision problems, these uncertainties can be complex and interrelated, with the result that the elicitation of precise mathematical measures such as probabilities become operationally difficult for decision makers to comprehend, and for facilitators to validate. In light of these obstacles, an alternative approach is to construct a number of narratives that describe possible ways in which the future might unfold. Each of these possible futures is conventionally termed a 'scenario' and the use of scenarios for strategic planning known as 'scenario planning' (e.g. Van der Heijden, 1996; Wack, 1985a, 1985b).

Advocates of scenario planning often prefer to avoid formal quantitative modeling (e.g. Schoemaker, 1995; Van der Heijden, 1996) and use informed but informal judgment – some examples can be found in Enserink (2000), Wollenberg, Edmunds, and Buck (2000), Cairns, Wright, Bradfield, van der Heijden, and Burt (2004). Nevertheless, efforts have been made to integrate decision analysis with the use of scenarios, as discussed for example in Goodwin and Wright (2009, chap.16) and Stewart (2005, chap. 11). The main objective of a scenario-based decision model is to use the philosophy of multi-criteria decision analysis (MCDA) to evaluate and compare the performances of alternatives in each scenario – given a decision problem, the approach considers that problem separately in each scenario before (possibly but not necessarily explicitly) combining this information to arrive at a final decision.

The general integration of scenario planning and MCDA has been thoroughly described in a number of places (e.g. Durbach & Stewart, 2012; Stewart, 2005, chap. 11; Stewart, French, & Rios, 2013), and a number of practical applications (using value function methods) reported (Montibeller, Gummer, & Tumidei, 2006; Ram, Montibeller, & Morton, 2010; van der Pas, Walker, Marchau, Van Wee, & Agusdinata, 2010). The current paper is narrower in scope and focuses only on the use of scenarios in the outranking methods, with the specific aims of:

1. Providing a formal description of the scenario-based outranking method;
2. Showing how the scenario-based outranking model provides a natural simplification to the comprehensive but practically complex "distributive" outranking method of d'Avignon and Vincke (1988);
3. Assessing the differences between results obtained using scenario-based and distributive methods, using a simulation experiment.

A primary motivation for this paper is the simplification of the distributive outranking method of d'Avignon and Vincke, which despite its theoretically appeal has not been widely used (indeed I could find no reported applications in the literature). By ignoring some aspects of uncertainty while assigning to others additional qualitative information, the scenario-based model avoids using

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measures that are potentially difficult for decision makers to interpret and work with (for example, stochastic indices of preference). Of course ignoring information brings with it the risk of selecting demonstrably worse alternatives, the extent of which is evaluated using a simulation experiment.

The remainder of the paper is structured as follows. In Section 2 notation is introduced and uncertainty modeling in the outranking methods reviewed. Section 3 describes the scenario-based outranking method and its relationship to the distributive method in d'Avignon and Vincke (1988). Section 4 clarifies the relationship with a brief numerical example. Sections 5 and 6 describe the simulation study and results respectively. A final section concludes the paper.

2. Outranking under uncertainty

Consider a decision problem consisting of I alternatives denoted by a_i, $i \in \{1, \ldots, I\}$, each evaluated on J criteria denoted by c_j, $j \in \{1, \ldots, J\}$. Let Z_{ij} be the evaluation of a_i in terms of criterion c_j, according to some suitable performance measure. Our concern is with decision making situations in which the values of Z_{ij} for each i are not known with certainty for all j, but are viewed as random variables with an associated multivariate probability distribution function $\mathbf{F}_i$ and probability density function $\mathbf{f}_i$. Let F_{ij} and f_{ij} denote the corresponding marginal cumulative distribution function and probability density function for criterion c_j if alternative a_i is selected.

Although F_{ij} and f_{ij} will usually be continuous functions, the range of possible outcomes can be approximated to arbitrary accuracy by a large number of discrete "states" or realizations of the associated random variable. Let $z_{ij,m}$ denote realization r_m of Z_{ij}, with $m \in \{1, \ldots, M_{ij}\}$. To explicitly differentiate between realizations/states and scenarios, performance in a particular scenario s_m is denoted by $z_{ij}^{(m)}$, with $m \in \{1, \ldots, S\}$. The set of constructed scenarios is denoted $\mathcal{S}$.

2.1. Methods using pairwise comparisons of probability distributions

Several outranking methods use stochastic dominance concepts to treat uncertain attribute evaluations. These categorize instances in which a pairwise comparison of the associated probability distributions is sufficient to confirm that one alternative is preferred to another (in the sense of maximizing expected utility) provided that certain constraints on the underlying utility function are satisfied. Well-known results (Bawa, 1975) show that the first-, second-, and third-degree stochastic dominance of a_i over a_k on c_j implies that a_i is preferred for, respectively: any increasing utility function on c_j; any concave increasing utility function on c_j; and any decreasingly risk averse, concave, increasing utility function on c_j. Similar conditions have been provided for convex utility functions (Goovaerts, de Vylder, & Haezendonck, 1984).

Zaras and Martel (1994) and Zaras (2001) use a simple weighted aggregation of indicator variables $\zeta_j(a_i, a_k)$ which equal 1 if a_i stochastically dominates a_k on criterion c_j and are otherwise zero. This results in a concordance index as for Electre I. Martel, d'Avignon, and Couillard (1986) and Azondékon and Martel (1999) construct a preference index $\zeta_j(a_i, a_k)$ as a product of three functions each scaled between 0 and 1 that cause $\zeta_j(a_i, a_k)$ to decrease as dominance weakens from first- to third-degree. A similar threshold-based method is provided in Nowak (2004). Dominance-based methods have also been extended to make use of fuzzy numbers, and possibilistic and evidentiary evaluations (Amor, Jabeur, & Martel, 2007; Boujelben, Smet, Frikha, & Chabchoub, 2009; Zaras, 2004), allowing for different uncertainty formats to be included in the same decision problem.

Other pairwise comparisons of probability distributions have been incorporated into stochastic outranking methods. Jacquet-Lagrèze (1977) allocates the part of f_{ij} (f_{kj}) where there is a non-zero probability of a_k (a_i) occurring as evidence in support of indifference, and then uses the cumulative distributions to allocate the remaining probability mass as evidence either that alternative a_i is preferred to a_k, or *vice versa*. Aggregation proceeds as for Electre I. A second set of models (Dendrou, Dendrou, & Houstis, 1980; Fan, Liu, & Feng, 2010; Liu, Fan, & Zhang, 2011; Martel et al., 1986) construct a matrix $\mathbf{P}^j$ whose entries P_{ik}^j denote the probability that alternative a_i is superior to alternative a_k on criterion c_j. The models differ with respect to the subsequent exploitation of these probabilities. Dendrou et al. (1980) and Liu et al. (2011) aggregate the P_{ik}^j using a weighted sum over attributes. Martel and d'Avignon (1982), Martel et al. (1986) use much the same approach but incorporate indifference and preference thresholds. Fan et al. (2010) compute joint probabilities associated with each of 2^J possible permutations of binary indicators denoting (attribute-specific) outranking between a pair of alternatives. Each of these is taken as evidence in favor of the 'superiority', 'inferiority', or 'indifference' of a_i relative to a_k, based on a user-defined threshold.

2.2. Distributive outranking methods

In all of the outranking models above the distributional aspect of the problem is fully absorbed into the problem at an early stage of the process through the definition of the P_{ik}^j or stochastic dominance relations. In contrast, (d'Avignon & Vincke, 1988) use the uncertain attribute evaluations to form a stochastic (or "distributive") outranking degree indicating the probability of attaining various degrees of outranking, rather than summarizing the stochastic evaluations directly as P_{ik}^j.

The first step involves defining a preference index $I_j(z_{ij,m}, z_{kj,n})$ denoting the degree of preference for $z_{ij,m}$ over $z_{kj,n}$. This index is scaled between 0 and w_j, where w_j is the normalized weight attached to criterion c_j. A random variable $H_j(a_i, a_k)$ can then be defined over the possible values of $I_j(z_{ij,m}, z_{kj,n})$, with the probability corresponding to each $H_j(a_i, a_k)$ given by

$$\Pr[H_j(a_i, a_k) = h] = \sum_{\{m,n:I_j(z_{ij,m}, z_{kj,n})=h\}} f_{ij}(z_{ij,m}) f_{kj}(z_{kj,n}) \qquad (1)$$

where $f_{ij}(z_{ij,m})$ denotes the probability of obtaining $z_{ij,m}$ for alternative a_i on criterion c_j.

The $H_j(a_i, a_k)$ can be aggregated into a distributive outranking degree $S(a_i, a_k)$ by addition over criteria *a la* Electre III or Promethee i.e. $S(a_i, a_k) = \sum_{j=1}^{J} H_j(a_i, a_k)$. Further random variables indicating average 'strengths' $S(a_i)$ and 'weaknesses' $W(a_i)$ are formed using unweighted averages (of $S(a_i, a_k)$ and $S(a_k, a_i)$ respectively, $\forall k \neq i$). The final exploitation of these measures to obtain a partial preference order is again complicated by the stochastic nature of the problem, with several suggestions (the simplest of which is to use the median of the distributions) proposed in d'Avignon and Vincke (1988).

Another distributive approach is to use Monte Carlo simulation from probability distributions. This is the approach followed by the outranking variant of the stochastic multi-criteria acceptability analysis (SMAA) method, SMAA-3 (Hokkanen, Lahdelma, Miettinen, & Salminen, 1998). SMAA is a family of inverse-preference methods that provides information about the types of preferences, if any, that would lead to the selection of each alternative. The approach simulates a large number of realizations from probability distributions governing (a) the uncertain attribute evaluations, and (b) parameters of the outranking model, and records the proportion and distinguishing features of those weights which result in each alternative obtaining a particular rank (often the "best"