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Decision Support

A proportional approach to claims problems with a guaranteed minimum

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ABSTRACT

In distribution problems, and specifically in bankruptcy issues, the *Proportional* (*P*) and the *Egalitarian* (*EA*) divisions are two of the most popular ways to resolve the conflict. Nonetheless, when using the egalitarian division, agents may receive more than her claim. We propose a compromise between the proportional and the egalitarian approaches by considering the restriction that no one receives more than her claim. We show that the *most egalitarian* compromise fulfilling this restriction ensures a minimum amount to each agent. We also show that this compromise can be interpreted as a process that works in two steps as follows: first, all agents receive an equal share up to the smallest claim if possible (egalitarian distribution), and then, the remaining estate (if any) is allocated proportionally to the remaining claims (proportional distribution). Finally, we obtain that the recursive application of this process finishes at the *Constrained Equal Awards* solution (*CEA*).

1. Introduction

A claims problem is a particular case of distribution problem, in which the amount to be distributed, the *estate E*, is not enough to cover the agents' claims on it. This model describes the situation faced by a court that has to distribute the net worth of a bankrupt firm among its creditors. But, it also corresponds with cost-sharing, taxation, or rationing problems. How should the scarce resources be allocated among its claimants? The formal analysis of situations like these, which originates in a seminal paper by O'Neill (1982), shows that a vast number of well-behaved solutions[1] have been defined for solving claims problems, being the *Proportional* and the *Equal Awards* (egalitarian) the two prominent concepts used in real world. The term well-behaved reflects the idea that the considered solutions might fulfill some principles of fairness, or appealing properties. A way of comparing solutions is given by the equity condition of Lorenz-dominance (see Dutta & Ray, 1989). A recent paper (Bosmans & Lauwers, 2011) compares the most usual bankruptcy rules in terms of Lorenz-dominance and analyzes those solutions that favor to smaller claimants relative to larger ones.

An illustrative example of claims problems is the fishing quotas reduction, in which the agent's claim can be understood as the previous captures, and the estate is the new (lower) level of joint captures. A similar example is given by milk quotas among the EU members.[2] In both examples, *proportionality* is the main principle used. Nevertheless, a minimal (*survival*) amount, guaranteed to each producer, should be fixed in order to ensure the profitability of fishing (milk) industries. That is, some part of the *estate* should be allocated in an *egalitarian way*. This idea is somewhat related to the axiom of *Sustainability* (see Herrero & Villar, 2002). As they mention,

> "Sustainability is a protective criterion for those agents with small claims. To illustrate this, consider the interpretation of a bankruptcy situation as a reduction in the fishing quotas. Here agent i's claim corresponds to her actual level of captures and the estate to be distributed to the new aggregate level of captures. Sustainable claims correspond to those levels of captures such that, if nobody else had a larger level, the aggregate new level of captures would not impose any rationing. Sustainability says that agents with sustainable claims should not be rationed after the change in the aggregate level of captures."

A similar situation can be found when a university distributes the budget to Departments. In this case, the resources are distributed proportionally to the number of Professors, students, subjects,

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[1] The reader is referred to Moulin (2002) and Thomson (2003) as surveys of this literature.

[2] Quotas were introduced in 1984. Each member state was given a reference quantity which was then allocated to individual producers. The initial quotas were not sufficiently restrictive as to remedy the surplus situation and so the quotas were cut in the late 1980s and early 1990s. Quotas will end on April 1, 2015.

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etc., but a minimal (fixed) amount is allocated to each regardless of size.

An alternative example of using the proportional approach is the way in which seats in the Spanish Parliament are allocated to each electoral district (province).[3] This is made proportionally to the population in each province, but a minimal number of seats (2) is guaranteed to each.[4] A similar situation is found in the US case: based on data from the decennial census, each state is allocated a proportion of the 435 seats in the United States House of Representatives, although each state is guaranteed a minimum of one seat, regardless of population.[5] The remaining seats are allocated one at a time, to the state with the highest priority number. This apportionment is based on the proportion of each state's population to that of the Fifty States together. We shall return to these examples later.

Although proportionality is the most used criterion,[6] whenever the smallest claim is very small compared with the largest one, a proportional division provides nearly nothing for this (these) small claimant(s). In this sense, the previous comments and examples show that real world, when applying proportional distributions, try to ensure an egalitarian (minimal) amount to each agent.

In this paper we will define a new solution concept that captures this behavior. This solution can be understood as a compromise between the proportional and the egalitarian distributions. In choosing this compromise, if we wanted to use the same weight on the proportional and the egalitarian distributions for each problem, the largest weight one could assign to the egalitarian distribution would be zero (otherwise for some problems an agent would receive an amount larger than her claim). So, we propose that the weight of each of the two distributions depends on the particular claims problem we are analyzing. In so doing, we define the weight used on the egalitarian distribution to be the highest weight such that the resulting vector satisfies the claims boundedness restriction.

Under an alternative view, we can differentiate between two different class of problems: the first class consists of problems where the per-capita estate is small relative to the smallest claim, $c_1 \geqslant E$ (a condition called in the literature as an *unsustainable* claim), whereas in the problems of the second class the smallest claim is *sustainable*. Then, if the claims problem is in the first category, the egalitarian distribution satisfies claims boundedness and all agents receive equal awards; if the claims problem falls in the second category, we first assign to each agent the smallest claim (egalitarian distribution), revise claims and estate accordingly, and then distribute the remaining estate proportionally to the revised claims (proportional solution). By this way, we define a new solution. Our main result, Proposition 3, shows that both approaches coincide in the same solution which we call $\alpha_{min} -$ *Egalitarian* solution.[7]

In short, our compromise solution:

- modifies the *Equal Awards* division, so that the proposal satisfies the claim-boundedness condition;
- modifies the *Proportional* division and considers a *minimal* amount that each agent should receive, which is endogenously determined in each particular problem (E,c);[8]
- provides a result that coincides with the one we would obtain if we assign to each agent this minimal amount, and distribute the remaining estate (if any) in a proportional way.

The paper is organized as follows: Section 2 contains the preliminaries. Section 3 presents our solution concept. Sections 4 and 5 provide the axiomatic analysis, and in Section 6 we present some final comments. The appendix gathers the proofs.

2. Preliminaries: claims problems

Throughout the paper we will consider a set of agents $N = \{1,2,\ldots,n\}$. Each agent is identified by her *claim*, c_i, $i \in N$, on the *estate* E. A **claims problem** appears whenever the estate is not enough to satisfy all the claims; that is, $\sum_{i=1}^n c_i > E$. Without loss of generality, we will order the agents according to their claims: $c_1 \leqslant c_2 \leqslant \cdots \leqslant c_n$. The pair (E,c) represents the claims problem, and we will denote by $\mathcal{B}$ the set of all claims problems. A *claims rule* (**solution**) is a single valued function $\varphi : \mathcal{B} \to \mathbb{R}^n_+$ such that, for each $i \in N$, $0 \leqslant \varphi_i(E,c) \leqslant c_i$, (**non-negativity** and **claim-boundedness**), and $\sum_{i=1}^n \varphi_i(E,c) = E$ (**efficiency**).

Many solution concepts have been defined in the literature aboutclaims problems (see for instance Thomson (2003) and Bosmans & Lauwers (2011)). The two most important criteria are the *Proportional* and the *Egalitarian* ones.

Definition 1. The *Proportional* solution, P. For each $(E,c) \in \mathcal{B}$ and each $i \in N$, $P_i(E,c) = \lambda c_i$, where $\lambda = \frac{E}{\sum_{i\in N} c_i}$.

Definition 2. The *Equal Awards* division, EA. For each $(E,c) \in \mathcal{B}$ and each $i \in N$, $EA_i(E,c) = \frac{E}{n}$.

It is easy to find examples in which the equal distribution of the *estate* exceeds some agent's claim.[9] In order to solve this situation the following modification of the EA division has been introduced.

Definition 3. The *Constrained Equal Awards* solution, CEA. For each $(E,c) \in \mathcal{B}$ and each $i \in N$, $CEA_i(E,c) \equiv \min\{c_i, \mu\}$, where μ is chosen so that $\sum_{i\in N} \min\{c_i, \mu\} = E$.

3. A proposal of solution: α_{min} – Egalitarian

Given the *Proportional* and the *Egalitarian* divisions, we consider now the family of compromises:

$$\varphi_\alpha = \alpha P + (1-\alpha)EA \quad \alpha \in [0,1].$$

That is, given a claims problem (E,c) involving n agents,

$$(\varphi_\alpha)_i(E,c) = \alpha \frac{c_i E}{\sum_{i=1}^n c_i} + (1-\alpha)\frac{E}{n} \quad \alpha \in [0,1].$$

The following example computes this proposal for several values of α.

[3] This example involves *indivisibilities*, which is not a trivial issue (see, for instance, Moulin (2000)).

[4] In the case of Spanish Parliament, the allocation mechanism is as follows (Spanish LOREG, 2011, art. 162): (1) Congress is composed of three hundred and fifty Deputies. (2) Each province has a corresponding initial minimum of two deputies. (3) The remaining two hundred and forty-eight deputies are distributed among the provinces in proportion to its population, according to the following procedure: (a) Obtain a distribution fee obtained by dividing by two hundred forty-eight the total number of the legal population of peninsular and island provinces. (b) Allocate to each province as many deputies as resulting, in whole numbers, dividing the population of provincial law by the quota allocation. (c) The remaining deputies are distributed by assigning one to each of the provinces whose quotient obtained under paragraph before, have a higher decimal fraction.

[5] "Each State shall have at Least one Representative" (U.S. Const., art. I, 2, cl. 3.).

[6] "In western society, for example, the customary solution would be to split the asset in proportion to the claims", see Young (1994, p. 123).

[7] An interesting question that has been addressed to us is if we can do the same for any claims rule ψ instead of the *Proportional* one. We will see that it is not possible, in general, to extend our results.

[8] We will see that our proposal satisfies a lower bound on awards property.

[9] For instance, consider the claims vector $c = (20,50,60)$ and the estate $E = 100$.