European Journal of Operational Research 232 (2014) 137–145

Contents lists available at SciVerse ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Computational Intelligence and Information Management

Allocation of information granularity in optimization and decision-making models: Towards building the foundations of Granular Computing

Witold Pedrycz *

Department of Electrical & Computer Engineering, University of Alberta, Edmonton, AB, Canada T6R 2V4
Warsaw School of Information Technology, Newelska 6, Warsaw, Poland

ARTICLE INFO

Article history:
Received 21 November 2011
Accepted 23 March 2012
Available online 30 March 2012

Keywords:
Granular Computing
Fuzzy sets
Allocation of information granularity

ABSTRACT

The highly diversified conceptual and algorithmic landscape of Granular Computing calls for the formation of sound fundamentals of the discipline, which cut across the diversity of formal frameworks (fuzzy sets, sets, rough sets) in which information granules are formed and processed. The study addresses this quest by introducing an idea of granular models – generalizations of numeric models that are formed as a result of an optimal allocation (distribution) of information granularity. Information granularity is regarded as a crucial design asset, which helps establish a better rapport of the resulting granular model with the system under modeling. A suite of modeling situations is elaborated on; they offer convincing examples behind the emergence of granular models. Pertinent problems showing how information granularity is distributed throughout the parameters of numeric functions (and resulting in granular mappings) are formulated as optimization tasks. A set of associated information granularity distribution protocols is discussed. We also provide a number of illustrative examples.

1. Introductory notes

Granular Computing [1–4,8,9,22] has emerged as a unified and coherent platform of constructing, describing, and processing information granules. The underlying concept of information granules has far reaching implications by giving rise to generic, semantically meaningful entities that are crucial to the perception of real world, modeling its phenomena and supporting decision-making processes. What has been said so far touched a qualitative aspect of the problem. The ongoing challenge is to develop a computing framework within which all these representation and processing endeavors could be formally realized. Granular Computing becomes innovative and intellectually proactive endeavor that manifests in several fundamental ways. It identifies the essential commonalities between the surprisingly diversified problems and technologies used there, which could be cast into a unified framework known as a granular world. This is a fully operational processing entity that interacts with the external world (that could be another granular or numeric world) by collecting necessary granular information and returning the outcomes of the Granular Computing. With the emergence of the unified framework of granular processing, we accomplish a better grasp as to the role of interaction between various formalisms and visualize a way in which they communicate. Granular Computing brings together the existing plethora of formalisms of set theory (interval analysis) [17], fuzzy sets [11,23–25], probabilistic sets [12,13] rough sets [6,16,18–20], shadowed sets [21] under the same roof by clearly visualizing that in spite of their visibly distinct underpinnings (and ensuing processing), they exhibit some fundamental commonalities. In this sense, Granular Computing establishes a stimulating environment of synergy among the individual approaches. By building upon the commonalities of the existing formal approaches, Granular Computing helps build heterogeneous and multifaceted models of processing of information granules by clearly recognizing the orthogonal nature of some of the existing and well established frameworks (say, probability theory coming with its probability density functions and fuzzy sets with their membership functions). Granular Computing fully acknowledges a notion of variable granularity whose range could cover detailed numeric entities and very abstract and general information granules. It looks at the aspects of compatibility of such information granules and ensuing communication mechanisms of the granular worlds. Granular Computing gives rise to processing that is less time demanding than the one required when dealing with detailed numeric processing [1]. Interestingly, the inception of information granules is highly motivated. We do not form information granules without reason. Information granules arise as an evident realization of the fundamental paradigm of abstraction.

* Address: Department of Electrical & Computer Engineering, University of Alberta, Edmonton, AB, Canada T6R 2V4. Tel.: +1 780 439 8731.

E-mail address: wpedrycz@ualberta.ca

0377-2217/$ - see front matter © 2012 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.ejor.2012.03.038

As underlined, the objective of Granular Computing is to create a unified view at all of the theoretical models of information granules and build on some essential similarities and subsequently arrive at fundamentals that help view this way of computing in broader context. These fundamentals are needed so that Granular Computing can effectively engage various formal frameworks it has been embracing for some time. One has to look at some general ways of forming information granules, irrespectively of a way they are formalized – a necessary prerequisite to start with their processing. The principle of justifiable granularity offers a certain viable alternative. Information granularity helps achieve better rapport with reality by bringing into a picture an issue of non-numeric data or results and quantifying its nature via information granules. This aspect is especially clearly visible in system modeling. There are no ideal models. The numeric, precise outcomes produced by models are not realistic. Information granularity has been engaged in one way or another in quantifying the lack of numeric precision. One admits a certain level of information granularity to make the model reflective of reality, quantify a limited knowledge about a phenomenon the model deals with, and capture the diversity of sources of knowledge and viewpoints articulated by individual decision-makers in processes of group decision-making [5,11]. The studies reported in [7,11,12] and carried out in the setting of rough sets offer an important and timely perspective at Granular Computing and dominance-based rough sets. Overall, information granularity can be then regarded as an essential design asset whose prudent usage becomes crucial to make models more realistic. This position gives rise to another fundamental principle of Granular Computing – an allocation of information granularity along with an optimization of the allocation process. In system modeling, an allocation of granularity elevates the existing models, no matter what their origin is, to a new level that could be referred to as *granular* models. As it will be discussed in the study, there are a number of convincing arguments that offer compelling evidence behind the emergence of granular models.

The objective of this study is to establish a concept of allocation of information granularity regarded as an important design asset in system modeling by giving rise to granular models. Along with the concept, discussed are protocols of allocation of information granularity throughout the system and the ensuing optimization. While there have been some discussions on granular models, especially those coming from the studies on fuzzy sets such as type-2 fuzzy models, controllers, and classifiers their design methodology has not been fully developed and in several cases there is a lack of motivation and convincing justification behind the constructs being introduced.

The material is organized in the following way. In Section 2, we highlight a transition from models to granular models and identify a number of compelling reasons behind the emergence of granular models. A formal problem statement is presented in Section 3; it is followed by a detailed discussion on formal models of information granules and their characterization in terms of information granularity (specificity), Section 4. The design of granular mappings is covered in Section 5 in which both a collection of protocols guiding an allocation of granularity as well as the underlying design criteria (optimization indexes) are studied. Fuzzy neural networks are used as a useful modeling example where we discuss the process of building granular fuzzy neural networks (Sections 6 and 7). Further formulations of allocation of information granularity are presented in Sections 8 and 9. A number of illustrative examples highlighting the main points of the constructs are distributed throughout the overall study.

In this study, a point of departure is a numeric mapping "*f*" from $\mathbf{R}^n$ to $\mathbf{R}$ of the form $f(\mathbf{x},\mathbf{a})$ where $\mathbf{a}$ is a p-dimensional vector of numeric parameters. The mapping itself could be realized in many different ways such as e.g., neural network, neurofuzzy system, rule-based system, and linear regression model. In this way our considerations exhibit a significant level of generality and are of relevance to a broad class of constructs.

2. From models to granular models

There are a number of interesting and practically legitimate design and application scenarios where the inherent granularity of the models becomes visible and plays an important role. We briefly highlight the main features of these modeling environments.

2.1. Granular characterization of models

It is needless to say that there are no ideal models which can capture the data without any modeling error meaning that the output of the model is equal to the output data for all inputs forming the training data. To quantify this lack of accuracy, we give up on the precise numeric model (no matter what particular format it could assume) and make the model granular by admitting granular parameters and allocating a predetermined level of granularity to the respective parameters so that the granular model obtained in this way "cover" as many training data as possible.

2.2. Emergence of granular models as a manifestation of transfer knowledge

Let us consider that for a current problem at hand we are provided with a very limited data set – some experimental evidence (data) **D** expressed in terms of input–output pairs. Given this small data, two possible scenarios could be envisioned:

(a) We can attempt to construct a model based on the data. As the current data set is very limited, designing a new model does not look quite feasible: it is very likely that the model cannot be constructed at all, or even if formed, the resulting construct could be of low quality.

(b) We would like to rely on the existing model (which although deals with not the same situation but has been formed on a large and quite representative body of experimental evidence. We may take advantage of the experience accumulated so far and augment it in a certain sense so that it becomes adjusted to the current quite limited albeit current data. In doing this, we fully acknowledge that the existing source of knowledge has to be taken with a big grain of salt and the outcomes of the model have to be reflective of partial relevance of the model in the current situation. We quantify this effect by making the parameters of the model granular (viz. more abstract and general) so that one can build the model around the conceptual skeleton provided so far. In this case, viewing the model obtained so far as a sound source of knowledge, we are concerned with a concept of an effective knowledge transfer, see Fig. 1. The

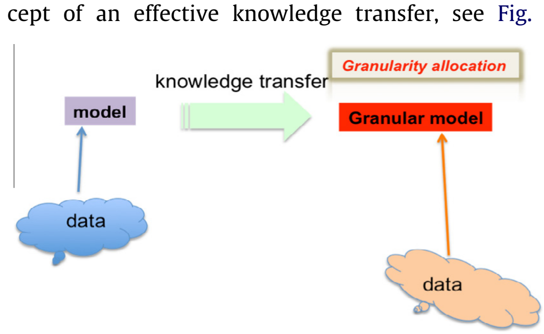

Fig. 1. Emergence of a granular model as a result of knowledge transfer.