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Strategic joining in M/M/1 retrial queues

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ABSTRACT

The equilibrium and socially optimal balking strategies are investigated for unobservable and observable single-server classical retrial queues. There is no waiting space in front of the server. If an arriving customer finds the server idle, he occupies the server immediately and leaves the system after service. Otherwise, if the server is found busy, the customer decides whether or not to enter a retrial pool with infinite capacity and becomes a repeated customer, based on observation of the system and the reward–cost structure imposed on the system. Accordingly, two cases with respect to different levels of information are studied and the corresponding Nash equilibrium and social optimization balking strategies for all customers are derived. Finally, we compare the equilibrium and optimal behavior regarding these two information levels through numerical examples.

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1. Introduction

Queueing systems in which arriving customers who find the server occupied may retry for service after a period of time are called retrial queues or queues with repeated orders. The retrial queueing system has been studied extensively due to its wide application. Apart from theoretical interests, it has been successfully applied to telephone switching systems, telecommunication networks and computer networks. The literature on retrial queueing systems is already very extensive, see Artalejo (1995), Artalejo and Gómez-Corral (1997, 1998), and Falin (2008). Interested readers are referred to Falin and Templeton (1997) and Artalejo and Gómez-Corral (2008) for extensive surveys on retrial queues.

During the last decades, considerable attentions have been devoted to the study of observable queueing systems which are concerned with customers' decentralized behavior and socially optimal strategies. Some reward–cost structures are imposed on the system to reflect the customers' desires for service and their unwillingness to wait. Customers are allowed to make decisions about their actions in the system. They want to maximize their benefits against the other customers who have the same objective, therefore the situation can be studied as a game among the customers. Such an economic analysis of queueing systems was pioneered by Naor (1969) with a single-server system with an observable queue, i.e., an arriving customer observes the number of customers and then makes his decision whether to join or balk. Edelson and Hildebrand (1975) considered the same queueing system with the assumptions that the customers make their decisions without being informed about the state of the system. Until now, there is a growing number of papers that deal with the economic analysis of the balking behavior of customers in variants of the M/M/1 queue, see Boudali and Economou (2012, 2007), Economou and Kanta (2008a,b), Economou and Manou (2012), Guo and Hassin (2011, 2012), Wang and Zhang (2011), among others. The monographs of Hassin and Haviv (2003) and Stidham (2009) summarized the main approaches and results from an economic viewpoint.

The majority of articles studied the economic analysis of the FCFS queues, in which a new customer is placed at the end of the queue, and therefore imposes no negative externalities on customers already in the system. However, when the service discipline in an observable queue is not FCFS, the decision of whether or not to join depends not only on the state of the queue but also on the strategy adopted by future arrivals. Hassin and Haviv (1997) dealt with the economic analysis of a queueing model with priorities in which two priority levels can be purchased and they obtained all of the Nash equilibrium strategies (pure or mixed) of the threshold type. Altman and Shimkin (1998) considered a system of observable egalitarian processor sharing (EPS) where customers decide whether to join the queue after observing the number of customers already there. They showed how to compute the unique (pure or mixed) Nash equilibrium point. Other related studies on the strategic behavior of customers in non-FCFS queueing systems include Adiri and Yechiali (1974), Tilt and Balachandran (1979), Hassin (1985), and Hassin and Haviv (2002).

In the retrial queueing system, every customer in the retrial pool repeats his demand until he receives his requested service. Therefore, the service discipline is not FCFS. The customers in the retrial pool are affected by the strategies adopted by future

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customers. There are some papers in the literature considered the strategic behavior of customers in queueing systems with retrials. But balking is forbidden. Kulkarni (1983a,b) investigated a single-server queueing system with two types of customers conducting retrials in which noncooperative and cooperative strategies were studied. Elcan (1994) derived socially optimal and equilibrium retrial rates for the case of single-server Markovian retrial queue with the classical retrial policy. Later, Hassin and Haviv (1996) extended the analysis in the case of general service times. Recently, Zhang et al. (2012) studied a single-server retrial queue with two types of customers in which the server is subject to vacations along with breakdowns and repairs. They discussed and compared the optimal and equilibrium retrial rates regarding the situations in which the customers are cooperative or noncooperative, respectively.

However, there are almost no articles dealing with equilibrium and socially optimal strategies in the retrial queue in which balking is allowed to the arriving customers. We aware of one exception. Economou and Kanta (2011) studied the retrial systems with the constant retrial policy and dealt with the equilibrium customer strategies and the social and profit maximization problems. But in their model, the customers in the retrial pool are served with FCFS discipline and the retrial rate is a constant.

In this paper, we first study customer equilibrium and socially optimal balking behavior in a classical retrial queue, where a customer arriving when the server accessible for him is busy leaves the service area but after some time repeats his demand. The paper assumes an exogenous retrial rate and the only decision is whether or not to join. Two cases will be distinguished with respect to the levels of information available to customers upon their arrivals, just before their decisions are made. Namely, (1) Unobservable case: Customers are informed only about the state of the server. (2) Observable case: Customers are informed the state of the server as well as the queue length in the retrial pool. We will give an extensive study of the corresponding Nash equilibrium and the socially optimal strategies for all customers in both cases. Finally, the effects of the main parameters on the equilibrium and optimal behavior of the customers, and the comparisons between the equilibrium and optimal strategies will be illustrated through numerical comparisons.

An example in practice of our model may be illustrated by the rational abandonment and decision making of customers in the call centers. In such service systems, modern call-center software system can provide a customer more or less information about the system upon arrival. A coming customer call will be served immediately if a server is available. If all servers are found busy, the customer will be put on hold, and may be asked to wait until a server becomes available. In some scenarios, the call center may choose to announce an expected waiting time to the customer at this point, while in some other situations the call center may provide no information. Customers have to make their decisions based on the possible information and estimate the economics of the decisions, taking the benefits associated with service completion and direct payments along with indirect costs associated with delays and retrials into account. To explore such information and its interpretation by customers, it is important to understand the impacts on system performance of different levels of information. For more details on the modeling and analysis of call centers with retrials and anticipated delays, we refer the reader to Aguir et al. (2004) and Whitt (1999), and references therein.

This paper is organized as follows. In the next section, we describe the dynamics of the model and the reward–cost structure. In Section 3, we determine Nash equilibrium and social maximizing strategies for joining the retrial pool in the unobservable case. In Section 4, we treat the corresponding observable case. In Section 5, some numerical examples are given. Finally, Section 6 provides the conclusions.

2. Description of the model

We consider a single server retrial queueing system in which primary customers arrive according to a Poisson process with rate λ. We assume that there is no waiting space, and if an arriving customer finds the server free, he immediately occupies the server and leaves the system after service completion. Otherwise, if the server is found busy, the customer goes into a retrial pool with infinite capacity and becomes a repeated customer. Each repeated customer repeats his demand after an exponential amount of time with rate θ, independently of other customers, until he receives his requested service, after which, he leaves and has no further effects on the system. The service times, for both primary and repeated customers, are exponentially distributed with parameter μ. We assume that the inter-arrival times of primary customers, intervals between repetitions and service times are mutually independent.

Denote by $I(t)$ the state of the server at time t. The events $I(t) = 0$ or 1 correspond to, respectively, the states that the server is idle or busy. Let $N(t)$ be the number of customers in the pool at time t. It is clear that the stochastic process $\{(I(t),N(t)),\ t \geqslant 0\}$ is a two-dimensional continuous time Markov chain with state space $\{0,1\} \times \{0,1,2,\ldots\}$. The non-zero transition rates are given by

$$q_{(0,i)(1,i)} = \lambda, \quad i = 0,1,2,\ldots;$$
$$q_{(1,i)(0,i)} = \mu, \quad i = 0,1,2,\ldots;$$
$$q_{(1,i)(1,i+1)} = \lambda, \quad i = 0,1,2,\ldots;$$
$$q_{(0,i)(1,i-1)} = i\theta, \quad i = 1,2,3,\ldots.$$

The corresponding transition rate diagram is shown in Fig. 1.

We assume that the customers are allowed to decide whether to join or balk upon their arrival based on the information they have. After service, every customer receives a reward of R units. This may reflect his satisfaction or the added value of being served. Moreover, each customer incurs a waiting cost C units per time unit when he remains in the system (this cost is accumulated both in the retrial pool and in the service area). We observe that the expected waiting cost incurred in the service area is $\frac{C}{\mu}$, independently of customers' strategy, and one can subtract it from R. Therefore, without loss of generality, we assume that waiting costs are incurred only for the time in the retrial pool. Customers are risk neutral and wish to maximize their expected benefit. At their arrival instants, they have to assess their expected waiting costs against their reward associated with receiving service, given the information regarding the system, and decide accordingly whether to join or not. Specifically, the customers strictly prefer to enter if the

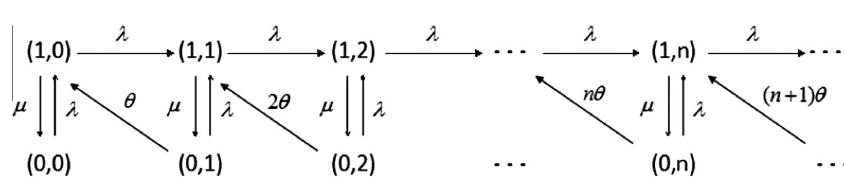

Fig. 1. Transition rate diagram of the original model with no balking.

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