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Decision Support

Robust weighted vertex p-center model considering uncertain data: An application to emergency management

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ABSTRACT

This paper presents a generalized weighted vertex *p*-center (WVPC) model that represents uncertain nodal weights and edge lengths using prescribed intervals or ranges. The objective of the robust WVPC (RWVPC) model is to locate *p* facilities on a given set of candidate sites so as to minimize worst-case deviation in maximum weighted distance from the optimal solution. The RWVPC model is well-suited for locating urgent relief distribution centers (URDCs) in an emergency logistics system responding to quick-onset natural disasters in which precise estimates of relief demands from affected areas and travel times between URDCs and affected areas are not available. To reduce the computational complexity of solving the model, this work proposes a theorem that facilitates identification of the worst-case scenario for a given set of facility locations. Since the problem is *NP*-hard, a heuristic framework is developed to efficiently obtain robust solutions. Then, a specific implementation of the framework, based on simulated annealing, is developed to conduct numerical experiments. Experimental results show that the proposed heuristic is effective and efficient in obtaining robust solutions. We also examine the impact of the degree of data uncertainty on the selected performance measures and the tradeoff between solution quality and robustness. Additionally, this work applies the proposed RWVPC model to a real-world instance based on a massive earthquake that hit central Taiwan on September 21, 1999.

1. Introduction

The *p*-center model, which aims to locate *p* facilities to minimize maximum distance (or travel time) between demand nodes and their closest facilities (e.g., Kariv and Hakimi, 1979; Albareda-Sambola et al., 2010), has been considered particularly suitable for emergency applications (e.g., Jia et al., 2007a,b; Huang et al., 2010), among various facility location models that have been presented in the literature (e.g., Mirchandani and Francis, 1990; Daskin, 1995; Altiparmak et al., 2006). This work adapts the *p*-center model to locate urgent relief distribution centers (URDCs) in an emergency logistics network that aims to promptly deliver relief supplies from URDCs to all relief or medical service stations in affected areas in the aftermath of quick-onset disasters (e.g., Altay and Green, 2006; Yi and Ozdamar, 2007; Campbell and Jones, 2011).

URDCs play an important role in an emergency logistics network, because they serve as hubs that seamlessly integrate and coordinate inbound and outbound emergence logistics responding to relief demands from affected areas. These hubs also have an inventory management function (i.e., risk pooling)—aggregating relief demands or their forecasts across several affected areas to reduce the adverse impact of relief demand variability and uncertainty on the system. In emergency response to quick-onset disasters, government rescue agencies typically designate existing public buildings (e.g., schools and stadiums) with little or no damage that can be promptly converted to shelters for survivors and/or warehouses for relief supplies as candidate sites of URDCs, instead of establishing new emergency facilities from scratch. Thus, the problem of locating URDCs can be considered as the vertex *p*-center problem which restricts the set of candidate sites to locations of existing public buildings (i.e., facility nodes). Furthermore, when relief demands faced by relief or medical stations are taken into account, the problem becomes a weighted vertex *p*-center (WVPC) problem (e.g., Current et al., 2002) with nodal weights reflecting relief demands from relief stations (i.e., demand nodes) in affected areas and the objective being to minimize maximum demand-weighted travel time between relief stations and their closest URDCs.

The proposed WVPC model for locating URDCs explicitly accounts for uncertain relief demands from relief stations and travel (or delivery) times between URDCs and relief stations, mainly due to poor measurements based on limited information available during a disaster's aftermath or approximations in the modeling process using aggregated demands and choosing a distance norm. Two major categories of approaches have been adopted in the literature to deal with uncertain coefficients in facility location models (Snyder, 2006), namely, stochastic programming (SP) and

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robust optimization (RO). The former has been used typically to deal with decision-making for facility locations in risk situations, in which the values of uncertain coefficients are governed by discrete or continuous probability distributions that are known to a decision-maker. The SP approach has been widely applied to emergency logistics for short-notice disasters (e.g., hurricanes, flooding, and wild fires) by assuming that possible impacts of these disasters can be estimated based on historical and meteorological data. The common goal of these stochastic location models is to optimize the expected value of a given objective function. A classical example of applying SP to disaster relief is the scenario-based, two-stage stochastic model proposed by Mete and Zabinsky (2010), for medical supply pre-positioning and distribution in emergency management. Other examples can be found in, for instance, Chang et al. (2007) and Balcik and Beamon (2008).

On the other hand, the RO approach attempts to optimize the worst-case system performance in uncertain situations that lack any information about the probability distributions of uncertain coefficients (e.g., Kouvelis and Yu, 1997); hence, the RO approach generally describes uncertain data using pre-specified intervals or ranges. Typical robustness measures include mini-max objective value and mini-max regret in an objective value. The RO approach may be more appropriate than the SP approach in emergency response to quick-onset or no-notice disasters (e.g., earthquakes, tsunamis, and landslides). For quick-onset disasters, because of the difficulty in predicting disaster occurrence and impacts as well as a lack of historical data, probability distributions and scenario data are generally unavailable. For example, an extremely large earthquake, 9.1 on the Richter scale, which hit the northeastern coast of Japan on March 11, 2011, was never considered in the nation's preparedness planning for earthquakes, even though Japan is widely regarded as one of the most advanced countries in earthquake preparedness. Thus, in responding to such a disaster, decision-makers may prefer an alternative method for describing uncertain data (i.e., using intervals to represent uncertain data).

In the proposed RWVPC model, uncertain relief demands at relief stations in affected areas and travel times between URDCs and relief stations are represented using prescribed intervals (or ranges), rather than probability distributions. The objective of locating p URDCs is to minimize worst-case deviation in maximum demand-weighted travel time between URDCs and relief stations from the optimal solution. This work proposes a theorem that facilitates identification of the worst-case scenario for a given set of URDC locations, thereby reducing complexity of solving the problem. Since the problem is *NP*-hard (Averbakh, 2003), a local search-based algorithmic framework incorporating the theorem for identifying the worst-case scenarios is developed to find robust solutions within a reasonable amount of computational resources. Then, a specific framework implementation based on simulated annealing (SA) is developed to conduct numerical experiments, including a case study based on the Jiji Earthquake, which hit central Taiwan on September 21, 1999.

The p-center problems with interval-represented uncertain data tend to be very difficult because of the mini-max structure. Therefore, analytical results and exact algorithms for the p-center problems with interval data have only been attained in special cases, such as locating a single facility on general networks or multiple facilities on tree networks (e.g., Averbakh and Berman, 2000; Burkard and Dollani, 2002). To the best of our knowledge, only Averbakh and Berman (1997) reported analytical results for an absolute weighted p-center problem with interval-represented nodal weights. No study has addressed absolute or vertex multi-center problems with both interval-represented edge lengths and nodal weights.

This study contributes significantly to the literature by (i) modeling the URDC location problem as the WVPC problem with interval-represented edge lengths and nodal weights on general networks; (ii) providing an effective and efficient algorithmic framework for solving the problem; and (iii) shedding light on the applicability and potential benefits of the proposed model to real-world instances.

The remainder of this paper is structured as follows. Section 2 describes the RWVPC problem, the representation of data uncertainty, and the property of worst-case scenarios. Section 3 presents the generic heuristic framework and a specific implementation using SA. This is followed by the numerical experiments in Section 4. Section 5 provides a case study demonstrating the applicability of the proposed model to real-world instances. Concluding remarks are given in Section 6.

2. Weighted vertex *p*-center problem with data uncertainty

2.1. The deterministic problem

Consider a connected, undirected network $G = (N, A)$, where N is the vertex set and A the arc (or edge) set. Let U be the set of candidate sites (i.e., facility nodes) for URDC locations and V be the set of relief stations (i.e., demand nodes) in affected areas; $U \cup V = N$, and $U \neq V$. Each possible pair of relief station $i \in V$ and URDC $j \in U$ is connected by an arc $(i, j) \in A$ that is associated with a positive (real or integer) number, t_{ij}, representing travel (or delivery) time between relief station i and URDC j. Although t_{ij} denotes the travel time, it can also be used for other measures of utility/disutility, such as distance or travel cost. Each relief station $i \in V$ faces relief demand ξ_i and is serviced by a single URDC. For a given set of predetermined candidate sites, U, the WVPC problem is to locate p ($p < |U|$) URDCs and assign relief stations to these centers, thereby minimizing maximum demand-weighted travel time between relief stations and URDCs. A mixed integer linear programming (MILP) formulation of the problem is as follows (e.g., Current et al., 2002).

$$\text{(WVPC)} \quad \text{Minimize } z, \tag{1}$$

$$\text{Subject to } z \geqslant \Sigma_{j\in U}\xi_i t_{ij} y_{ij}, \quad \forall i \in V, \tag{2}$$

$$\Sigma_{j\in U} y_{ij} = 1, \quad \forall i \in V, \tag{3}$$

$$y_{ij} - x_j \leqslant 0, \quad \forall i \in V, \quad j \in U, \tag{4}$$

$$\Sigma_{j\in U} x_j = p, \tag{5}$$

$$x_j \in \{0, 1\}, \quad \forall j \in U, \tag{6}$$

$$y_{ij} \in \{0, 1\}, \quad \forall i \in V, \quad j \in U. \tag{7}$$

The decision variables are binary variables x_j, $\forall j \in U$ and y_{ij}, $\forall i \in V$, $j \in U$. $x_j = 1$ if candidate site j is selected; otherwise, $x_j = 0$. Additionally, $y_{ij} = 1$ if relief station i is serviced by URDC j; otherwise, $y_{ij} = 0$. The objective function (1) minimizes maximum demand-weighted travel time between relief stations and URDCs. Constraint (2) defines the lower bound of maximum demand-weighted travel time, which is being minimized. Constraint (3) requires that each relief station be assigned to exactly one URDC. Constraint (4) restricts relief station assignments only to selected URDCs. Constraint (5) stipulates that p URDCs are to be located. Constraints (6) and (7) indicate that location and allocation decision variables are binary. The WVPC problem is also known as the minimum k-supplier problem (Ausiello et al., 1999).

2.2. Representation of data uncertainty and the robust WVPC problem

Uncertain relief demands at relief stations and travel times between relief stations and URDCs are described using intervals or ranges. Specifically, An interval $[\xi l_i, \xi u_i]$, $0 \leqslant \xi l_i < \xi u_i$, represents uncertain relief demand at station i, and an interval $[tl_{ij}, tu_{ij}]$, $0 \leqslant tl_{ij} < tu_{ij}$, captures the uncertainty of travel time between station i and URDC j. Let W be the Cartesian product of intervals $[\xi l_i, \xi u_i]$,