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Decision Support

Super efficiencies or super inefficiencies? Insights from a joint computation model for slacks-based measures in DEA

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ABSTRACT

The slacks-based measure (SBM) can incorporate input and output slacks that would otherwise be neglected in the classical DEA model. In parallel, the super-efficiency model for SBM (S-SBM) has been developed for the purpose of ranking SBM efficient decision-making units (DMUs). When implementing SBM in conjunction with S-SBM, however, several issues can arise. First, unlike the standard super-efficiency model, S-SBM can only solve for super-efficiency scores but not SBM scores. Second, the S-SBM model may result in weakly efficient reference points. Third, the S-SBM and SBM scores for certain DMUs may be discontinuous with a perturbation to their inputs and outputs, making it hard to interpret and justify the scores in applications and the efficiency scores may be sensitive to small changes/errors in data. Due to this discontinuity, the S-SBM model may overestimate the super-efficiency score. This paper extends the existing SBM approaches and develops a joint model (J-SBM) that addresses the above issues; namely, the J-SBM model can (1) simultaneously compute SBM scores for inefficient DMUs and super-efficiency for efficient DMUs, (2) guarantee the reference points generated by the joint model are Pareto-efficient, and (3) the J-SBM scores of a firm are continuous in the input and output space. Interestingly, the radial DEA efficiency and super-efficiency scores for a DMU are continuous in the input–output space. The J-SBM model combines the merits of the radial and SBM models (i.e., continuity and Pareto-efficiency).
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1. Introduction

Over the past decades, enormous progress in both theories and applications has been made in the field of data envelopment analysis (DEA). One key assumption of the classical DEA model is that a firm's relative efficiency among its peers depends on how much it can proportionally expand all of its outputs given its inputs (or reducing all of its inputs given its outputs) under the technological constraint. These models are usually called "radial" or "Farrel-type" efficiency models (Farrell, 1954), and most DEA models developed since the seminal work of Charnes et al. (1978) belong to this category. However, the classical "radial" DEA model has been known for having two primary limitations. First, some decision-making units (DMUs) may be measured against a weakly efficient input–output point in the production possibility set. These weakly efficient points, which also serve as the *reference points* for their corresponding DMUs, have positive input or output slacks with respect to strongly Pareto-efficient points on the efficient frontier. Second, a high proportion of DMUs can turn out to be efficient.[1] If a total ordering among the efficient DMUs is desired, one may impose an exogenously determined preference structure for inputs or outputs; e.g., restrictions on the dual multipliers or preferential weights; see Angulo-Meza and Lins (2002) for a review. Alternatively, one may also use the super-efficiency model (Anderson and Perterson, 1993), which does not require a prior weight assignment as in the DEA models with weight restrictions.

In light of the issue of referencing non-Pareto-efficient targets, Tone (2001) proposes a novel slacks-based measure (SBM) to eliminate slacks-related biases in efficiency measurement. The hallmark of SBM is that the SBM efficiency score is a function of input and output slacks. The idea behind SBM then forms a sharp contrast with the classical radial efficiency measure, in which efficiency is determined based on either equi-proportional input-contraction or output-expansion. One salient advantage of SBM is that the SBM model is guaranteed to identify a Pareto-efficient reference point for the evaluated DMU. Therefore SBM resolves the "slacks" issue found to exist in the radial DEA models, but results from the SBM model may still contain a high proportion of efficient DMUs. Tone (2002) develops a super-efficiency model for SBM (S-SBM) to further compare the performance of efficient DMUs. The S-SBM model draws on a modeling concept similar to the one first put forward in Anderson and Peterson (1993), in which each DMU is evaluated against the efficiency frontier that

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[1] A main contributing factor to this problem is due to a small sample size relative to the number of inputs and outputs. See e.g., Dyson et al. (2001). Yet this degeneracy issue is not one that is exclusive to the radial model, but more of a general limitation shared by both radial and non-radial models (Cook and Seiford, 2009).

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one would obtain if the DMU under evaluation is removed from the sample.

In this paper, I highlight three major implementation issues for the S-SBM model, and I propose a new model to simultaneously tackle these issues. The three issues are described as follows. First, the S-SBM model can only be used to compute super-efficiency scores for efficient DMUs, but not SBM scores for inefficient DMUs. Specifically, obtaining SBM scores and S-SBM scores for efficient DMUs requires applying both SBM and S-SBM models: first we need to calculate SBM scores for all observations, then single out efficient DMUs according to the SBM scores, and finally apply S-SBM models to these efficient DMUs to calculate their S-SBM scores. These three steps can be cumbersome to implement in practice, especially for large scale application involving multiple sample groups, for example, when computing the SBM Malmquist productivity index over a long panel; see Tone (2004). By contrast, the super-efficiency model for the radial DEA model by Anderson and Peterson (1993) can be used to compute both DEA efficiency and super-efficiency scores, saving the need for switching back and forth between the efficiency and super-efficiency models. Second, the S-SBM model may generate weakly efficient reference points, which are at odds with the original notion that motivates the development of SBM; namely, slacks must be taken into account and the influence be removed from efficiency assessment. Finally, I discover that there exists a discontinuous gap between the SBM and S-SBM scores of a weakly efficient DMU when it is subject to small perturbations of input–output data. This discontinuity or gap between the SBM and S-SBM scores may lead to great difficulty in interpreting the scores because of the sensitivity to small measurement errors or noise in the data. That is, an S-SBM efficient observation may become extremely SBM inefficient upon a small increase in inputs or a small decrease in output (and vice versa). As a result, the S-SBM score in this case is an overestimate of super-efficiency.

This paper proposes an ambidextrous model for computing both SBM and S-SBM efficiencies respectively for inefficient and efficient DMUs. Reference points in the proposed joint model (J-SBM) are Pareto-efficient, and the J-SBM scores are continuous in input–output data. One key challenge of building the joint model is to embed a toggle under which the joint model can function as a SBM model when the evaluated unit is inefficient, and as an S-SBM model when the evaluated observation is efficient. This is achieved by formulating the joint model as a mixed-integer linear programming problem. The J-SBM model simplifies the computational procedure for computing SBM and S-SBM efficiency scores, and at the same time incorporates slacks that would otherwise be unaccounted for in the existing super-efficiency model for SBM.

The paper proceeds as follows. Formulations of SBM and S-SBM and their limitations are studied in the Section 2; then I propose a joint model for SBM and S-SBM to overcome these limitations. In Section 3 I use a numerical example to compare the joint model with the existing ones. Section 4 provides a summary of contribution and discussion of the findings.

2. Mathematical formulation

2.1. Slacks-based measure for technical efficiency

Let us first introduce the notation used in this paper. Suppose n DMUs are observed, and DMU j ($j = 1,\ldots,n$) uses inputs $X_j = (x_{j1},\ldots,x_{jm}) \in \Re^m_+$ to produce outputs $Y_j = (y_{j1},\ldots,y_{js}) \in \Re^s_+$. The slacks-based measure (SBM) for, say DMU k, is defined to be the optimal value of the following problem:

$$
\begin{aligned}
\min \quad & \rho_k = \left(\frac{1 - \frac{1}{m}\sum_{i=1}^{m} s_i^- / x_{ki}}{1 + \sum_{r=1}^{s} s_r^+ / y_{kr}} \right) \\
\text{Subject to} \quad & \sum_{j=1}^{n} \lambda_j x_{ji} = x_{ki} - s_i^- \text{ for } i = 1,\ldots,m \\
& \sum_{j=1}^{n} \lambda_j y_{jr} = y_{kr} + s_r^+ \text{ for } r = 1,\ldots,s \\
& \lambda_j \geqslant 0 \text{ for } j = 1,\ldots,n \\
& s_i^- \geqslant 0 \text{ for } i = 1,\ldots,m \\
& s_r^+ \geqslant 0 \text{ for } r = 1,\ldots,s
\end{aligned} \tag{1}
$$

The left-hand-sides of the constraints in Model (1) consist of non-negative linear combinations of all observed DMUs, which form the efficient frontier of the model. The inputs and outputs of DMU k are bounded by the frontier on left-hand-sides of the constraints. The variable λ_j indicates the intensity under which DMU j takes part in forming the efficient frontier. The constraints in Model (1) are similar to those in the classical DEA model (Charnes et al., 1978). What is different is that the efficiency measure (i.e., the objective function) in SBM is a function of input and output slacks; so, compare with the standard DEA model, SBM is non-oriented, non-radial, and always identify Pareto-efficient reference points (I will give a more precise definition of reference points shortly.). The SBM efficiency score is bounded between zero and one (Tone, 2001): efficient DMUs are those having an efficiency score of one (i.e., all slacks are zero), and the lower the value, the lower the relative efficiency.

As the base model has been introduced, from this point on I will use a superscript "$*$" to denote the optimal solution value of a variable in the SBM model. Now it is appropriate to define Pareto-efficiency:

Definition 1 (*Pareto-efficiency; SBM efficiency*). A DMU is Pareto-efficient if and only if in optimality $s_i^{-*} = s_r^{+*} = 0\ \forall i$ in Model (1).

The idea of Pareto-efficiency is equivalent to that of SBM. A DMU is Pareto-efficient if it is impossible to make unilateral improvement on any inputs or outputs. Pareto-efficiency is also tied to the two-phase approach described earlier: if a DMU is deemed efficient in Phase-I and has its optimal slack values all zero in the additive model in Phase-II, the DMU is also Pareto-efficient (Cooper et al., 2007, pp. 43-46).

Observe that Model (1) is a fractional linear programming problem. The Charnes–Cooper transformation can be used to transform it into a linear programming problem (Charnes and Cooper, 1962; Charnes et al., 1978; Tone, 2001), which can then be solved by efficient algorithms:

$$
\begin{aligned}
\min \quad & \rho_k = t - \frac{1}{m}\sum_{i=1}^{m} s_i^- \Big/ x_{ki} \\
\text{Subject to} \quad & t + \frac{1}{s}\sum_{r=1}^{s} s_r^+ \Big/ y_{kr} = 1 \\
& \sum_{j=1}^{n} \lambda_j x_{ji} = t x_{ki} - s_i^- \text{ for } i = 1,\ldots,m \\
& \sum_{j=1}^{n} \lambda_j y_{jr} = t y_{kr} + s_r^+ \text{ for } r = 1,\ldots,s \\
& \lambda_j \geqslant 0 \text{ for } j = 1,\ldots,n \\
& s_i^- \geqslant 0 \text{ for } i = 1,\ldots,m \\
& s_r^+ \geqslant 0 \text{ for } r = 1,\ldots,s \\
& t \geqslant 0
\end{aligned} \tag{2}
$$

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