European Journal of Operational Research 225 (2013) 324–331

Contents lists available at SciVerse ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Decision Support

Structural properties of Markov modulated revenue management problems

Can Özkan, Fikri Karaesmen *, Süleyman Özekici

Department of Industrial Engineering, Koç University, 34450 Sarıyer-Istanbul, Turkey

ARTICLE INFO

Article history:
Received 17 November 2011
Accepted 13 September 2012
Available online 3 October 2012

Keywords:
Revenue management
Dynamic programming
Markov modulation

ABSTRACT

The admission decision is one of the fundamental categories of demand-management decisions. In the dynamic model of the single-resource capacity control problem, the distribution of demand does not explicitly depend on external conditions. However, in reality, demand may depend on the current external environment which represents the prevailing economic, financial, social or other factors that affect customer behavior. We formulate a Markov Decision Process (MDP) to maximize expected revenues over a finite horizon that explicitly models the current environment. We derive some structural results of the optimal admission policy, including the existence of an environment-dependent thresholds and a comparison of threshold levels in different environments. We also present some computational results which illustrate these structural properties. Finally, we extend some of the results to a related dynamic pricing formulation.

1. Introduction

Revenue management is a field that originates in the Airline Deregulation Act of 1978 (Talluri and van Ryzin, 2004a). There have been many studies since 1978 on different aspects of revenue management. Detailed overviews can be found in Talluri and van Ryzin (2004a) and Chiang et al. (2007). An important building block model for more complicated revenue management is single resource capacity control. It is common in airline companies to sell identical seats at different fares. The major issue is the decision process of accepting or rejecting a booking request of a certain class for a given resource. The static model in which different fare classes arrive at different, nonoverlapping time stages ordered in an increasing fare class rewards, is first considered by Littlewood (1972). The dynamic programming model of this problem is analyzed by Lee and Hersh (1993), and the structure of the optimality policy is investigated by Lautenbacher and Stidham (1999). For further research on single resource capacity control, see Brumelle and McGill (1993), Talluri and van Ryzin (2004b), Barz and Waldmann (2007), Lan et al. (2008), Birbil et al. (2009), and Aydın et al. (2009).

Many sophisticated models exist for the single resource problem in the revenue management literature. Most of these models assume that the arrival process of fare classes is independent of external factors that may be varying randomly over the planning horizon. On the other hand, there are situations where the demand rate is strongly dependent on some external process, which we call the environmental process. We model this environmental process through a Markov chain. We consider the single resource capacity control problem in revenue management in such fluctuating demand environments. We refer to the corresponding model as Markov-modulated single resource capacity control. Such a model has not been discussed widely in a revenue management context. To our knowledge, the only study that explicitly models this situation is Chapter 4 of Barz (2007). In that chapter, Barz considers an environmental process for a single-resource control problem under very general assumptions. In particular, her model considers an infinite horizon problem with possibly random planning horizons. She shows that the optimal admission policy must be of threshold type for this generic model. On the other hand, the complexity of her model prevents further structural results on the effects of time, environments and other relevant model parameters. To investigate these properties, we consider a discrete-time finite horizon problem that is less general than that of Barz but otherwise follows the standard assumptions with respect to the general literature. This model enables us to investigate time-related properties of the optimal policy and the effects of external environments. Moreover, for this model, we can also analyze the effects of varying problem parameters such as arrival rates, rewards and the transition matrix of the environmental process. This not only extends the results of Aydın et al. (2009) to a more general setting but also allows comparing optimal policies in different environments and for varying environment process parameters. Overall, this analysis presents a complete picture for this problem.

Even though the fluctuating demand environment is little studied in revenue management, there is a significant number of papers related to Markov-modulated models for inventory systems, see Song and Zipkin (1993), Özekici and Parlar (1999), Arifoğlu and Özekici (2010), Gayon et al. (2009). For example, Song and Zipkin

* Corresponding author. Tel.: +90 212 338 1718; fax: +90 212 338 1548.
E-mail addresses: canozkan@ku.edu.tr (C. Özkan), fkaraesmen@ku.edu.tr (F. Karaesmen), sozekici@ku.edu.tr (S. Özekici).

0377-2217/$ - see front matter © 2012 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.ejor.2012.09.020

(1993) argue that demand frequently depends on external factors which they call the current state of the world. They also argue that this current state of the world can be described by factors based on economic, financial and other conditions. van Ryzin (2005) emphasizes the needs for better demand modeling for revenue management. In particular, he mentions that standard demand models in revenue management treat causal variations based on external factors as noise. The environment-based framework addresses this issue. van Ryzin (2005) points out short term market conditions as a significant factor. These include competitors' availabilities and prices. In addition, there is evidence that the aggregate demand is affected by external market forces such as currency exchange rates and energy prices. Finally, weather conditions, such as forecasted snow storms or heat waves are important short term external factors that are known to impact demand in hotel and airline revenue management. Although these external factors seem to be very different from each other, they all influence the demand. This motivates the need for modeling the effects of such factors through an environment-dependent demand model. Finally, there is reason to believe that environment-based demand may have a bigger impact in revenue management problems than in inventory replenishment problems. In inventory replenishment, the ordering decision helps absorbing some of the variability in demand. But in revenue management there is typically no replenishment opportunity and demand variability has to be addressed only by admission or pricing decisions.

The rest of the paper is organized as follows. We first provide the model notation and formulation in Section 2. Then, we identify some structural properties of the optimal admission control policy in Section 3. We analyze the effects of varying problem parameters in Section 4. In Section 5 we provide an example to illustrate our structural results and asses the benefits of using an environment based model. Next, we extend the idea of a fluctuating demand environment to a dynamic pricing problem in Section 6. Finally, in Section 7 we conclude the paper. Most of the technical details involving proofs and derivations are relegated to the Appendix.

2. Model formulation

We formulate a discrete time, finite horizon (T periods) MDP model of the admission control problem corresponding to single-leg capacity control.

Let $E_t \in \{1, 2, \cdots, M\}$ denote the randomly fluctuating external environment. $E = \{E_0, E_1, \cdots, E_T\}$ is assumed to be a Markov chain with transition matrix $\mathbf{P}$ where $p_{ij} = P\{E_{t+1} = j | E_t = i\}$. We assume that there is at most one arrival and that each arrival from a fare class can request a finite number of seats in each stage. The probability that fare class a arrives at any stage is denoted by r_{ja} when the current environment is j. The probability of no arrival in a given environment is denoted by r_{j0}. Therefore, $\sum_{a=0}^{N} r_{ja} = 1$ for any j. Non-stationary demand scenarios can be handled by defining appropriate environment and transition matrices. For each fare class a, suppose there is an upper bound B_a on the number of fare products requested. Let q_{jab} denote the probability that b units of inventory is requested given that current environment is j and the requested fare class is a.

In each stage t, the firm must choose the optimal number of seats to be sold for each fare class. We assume that customers accept the scenario of a partial satisfaction of their request. Brumelle and Walczak (2003) showed that structural results on the optimal policy are not valid in case of acceptance or rejection of the whole demand when there is no environment process (Also, see Van Slyke and Young (2000) and Çil et al. (2007) for related issues). Therefore, we only analyze the case where customers accept the partial satisfaction of their requests. For each sold ticket, the reward is $c(a)$ if the fare class is a. The transition probabilities and reward function are assumed to be stationary and we suppose that the fare classes are ordered so that $c(a_1) \leqslant c(a_2)$ when $a_1 \leqslant a_2$. We let $\mathbb{Z}_+$ denote the set of positive integers and $\mathbb{R}$ denote the set of real numbers.

We also use the following notations:

$v_t(x, j)$	= expected maximum revenue from period t on, given that current inventory level is x and environment is j.
$\Delta v_t(x, j)$	$= v_t(x, j) - v_t(x-1, j)$
$(x)^+$	$= \max\{x, 0\}$
$U(b, x)$	$= \{0, 1, \cdots, \min\{b, x\}\}$

The optimal expected revenue and the admission control policy for this problem can be obtained by solving the following Bellman equation

$$v_t(x,j) = \sum_{a=1}^{N} r_{ja} \sum_{b=1}^{B_a} q_{jab} \max_{u \in U(b,x)} \left\{ \sum_{k=1}^{M} p_{jk} v_{t+1}(x-u,k) + c(a)u \right\} + r_{j0} \sum_{k=1}^{M} p_{jk} v_{t+1}(x,k), \tag{1}$$

with boundary conditions

$v_t(0,j) = 0$ for $j = 1, 2, \ldots, M$.
$v_T(x,j) = 0$ for any $x \in \mathbb{Z}_+$ and $j = 1, 2, \ldots, M$.

For obtaining structural results, the following equivalent representation that uses the definition of Δv_t turns out to be helpful

$$\begin{aligned} v_t(x,j) &= \sum_{a=1}^{N} r_{ja} \sum_{b=1}^{B_a} q_{jab} \max_{u \in U(b,x)} \\ &\left\{ c(a)u - \sum_{k=1}^{M} p_{jk} \left(\sum_{z=1}^{u} \Delta v_{t+1}(x+1-z,k) - v_{t+1}(x,k) \right) \right\} \\ &+ r_{j0} \sum_{k=1}^{M} p_{jk} v_{t+1}(x,k) = \sum_{a=1}^{N} r_{ja} \sum_{b=1}^{B_a} q_{jab} \max_{u \in U(b,x)} \\ &\left\{ \sum_{z=1}^{u} \left(c(a) - \sum_{k=1}^{M} p_{jk} \Delta v_{t+1}(x+1-z,k) \right) \right\} + \sum_{k=1}^{M} p_{jk} v_{t+1}(x,k), \end{aligned} \tag{2}$$

where the sum is set to be zero when $u = 0$.

3. Structural properties

In this section, we investigate some structural properties of the Markov-modulated single-resource capacity control problem. To begin with, it is intuitive that if we have one more inventory, then expected revenue should be larger. Similarly, expected revenue should be larger if we have more time to go. These claims can be easily proven by induction on t. Second order properties are less trivial. In the following theorem, we establish the concavity of $v_t(x, j)$ in x.

Theorem 1. *$v_t(x, j)$ is a concave function in x for any environment j and time t.*

We provide the proofs of this section in Appendix A. Theorem 1 establishes that $\Delta v_t(x, j)$ decreases as we increase the inventory level x. By considering (2), we can conclude that $c(a) - \sum_{k=1}^{M} p_{jk} \Delta v_{t+1}(x+1-z,k)$ is decreasing in z. Therefore, in (2) we should increase $u \leqslant \min\{b, x\}$ until $c(a) - \sum_{k=1}^{M} p_{jk} \Delta v_{t+1}(x+1-z,k)$ becomes negative or u is equal to $\min\{b, x\}$. Since