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Innovative Applications of O.R.

A modified Duckworth–Lewis method for adjusting targets in interrupted limited overs cricket

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ABSTRACT

In this paper we present a modified Duckworth/Lewis method. The key modification is an improved functional form for the model describing the runs to be scored in an innings. In the course of our work we compare several alternative methods for resetting targets in limited overs cricket that have been proposed in the literature and conclude that the Duckworth/Lewis method is the most viable. Our analysis also suggests that it is reasonable to use a single method for both the 50-over and 20-over formats of the game.

1. Introduction

Cricket is a hugely popular sport around the world. An estimated three billion people are cricket fans, a figure that is larger only for soccer, which has an estimated 3.5 billion fans (www.digalist.com). In recent years cricket's governing body, the International Cricket Council (ICC), has sought to make cricket even more popular. In order to achieve this, one strategy the ICC has adopted is to introduce Twenty20 (T20), a shorter format of the game, with the intention of making cricket a faster, more exciting spectacle that might attract a new audience.

Broadly speaking cricket can be played in two formats: limited overs and non-limited overs games. Non-limited overs matches at the professional level typically last for several days. For example, in the case of international games between major cricket playing countries, a 'test match' lasts for five days. Limited overs matches on the other hand, are designed to start and finish on the same day. For example, One Day Internationals (ODI) are limited to 50 overs per side, whilst T20 matches are limited to 20 overs per side and are the shortest format of international cricket, with matches typically lasting for three hours, bringing the game closer to the time span of other popular spectator sports.

In comparison to other sports, limited overs cricket is particularly vulnerable to inclement weather – when it rains, or becomes too dark, cricket becomes too dangerous to play. As a consequence, when a ODI or T20 match is interrupted by rain or bad light, either or both of the competing teams can often not complete their allotted overs. Incomplete games are unsatisfactory for the players and fans alike and, to some extent negate the purpose of the shorter formats since an abandoned match offers minimal levels of excitement. Furthermore, to enable knockout tournament play, such as the ODI and T20 World Cups, games must reach a conclusion. Therefore, the cricket authorities have adopted quantitative methods to adjust scores and reset targets in matches when one or both sides cannot complete their allotted overs in order to ensure interrupted matches are concluded and a definite result is obtained.

Since the first limited overs match was played in 1962, cricket analysts have searched for a fair method to reset targets in interrupted matches. The issue was elevated to higher importance following the introduction of the ODI World Cup in 1975. Several methods have been tried by the ICC. The current method, the Duckworth-Lewis (D/L) method (Duckworth and Lewis, 1998) is now widely accepted as the fairest method available and has been in operation since 1997.

Several academic papers have appeared attempting to improve upon the D/L method and these can be split into two categories: resources based methods and probability-preserving based methods. Possibly the highest profile alternative is the VJD method of Jayadevan (2002) which can be interpreted in terms of resources. Stern (2009) proposes changing the resources table of the D/L method in the second innings to better reflect how teams batting second are able to adopt a different strategy from the team batting first. Bhattacharya et al. (2011) present an alternative resources table for the D/L method based on a non-parametric approach for T20 cricket. Preston and Thomas (2002) were the first authors to present a method for adjusting targets that preserves the probability of

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victory for each team as it stood before the interruption took place. Carter and Guthrie (2004) follow a similar ethos and present algorithms to preserve the probability of victory for several interruptions during a game. We do not consider a probability-conservation method as this methodology was found to give contradictory results in Duckworth and Lewis (2005). Further, it has never been adopted by the ICC and so we choose to seek improvements to the current methodology.

In this paper we first present a method for estimating the D/L model parameters and then show that the D/L method is superior to other proposed resources-based methods for resetting targets in cricket. We then examine the scoring patterns in T20 Internationals (T20Is) and the last 20 over of ODIs and conclude that, as Duckworth and Lewis do, it is reasonable to use a single model for both ODI and T20I formats. Next, we present a modified D/L method which provides a superior fit to data and proves to better reflect the run scoring pattern observed in limited overs cricket. Finally we present an empirical justification for the adjustment of our modified D/L model in the case of high scoring matches. In Section 2 we present the current D/L method and a method to estimate the parameters of the model. In Section 3 we describe our data and compare the runs scoring pattern of the two formats, ODI and T20I. Comparison of the D/L method with other resources-based methods for resetting targets is presented in Section 4. Section 5 describes our modified D/L model and we conclude with some closing remarks in Section 6.

2. The D/L method

The D/L method has been through two incarnations. The first was adopted by the ICC in 1997 and is described in Duckworth and Lewis (1998). The second version, known as the Professional Edition, was introduced in 2003 (see Duckworth and Lewis, 2004) so that the method produced fairer adjusted targets in high scoring interrupted games.

The fundamental idea behind the D/L method is to estimate the resources available, R, to each team. In an uninterrupted match each team will have 100% of its resources available and no target adjustment is necessary. However, if there is an interruption and the resources of team 1, R_1, are not equal to team 2's resources, R_2, then the target for team 2 must be adjusted. Let S be the total runs scored by team 1 (the team batting first), then the D/L method states that the par score for team 2 (the team batting second) T, is given by

$$\begin{aligned} T &= SR_2/R_1 & \text{if } R_2 \leqslant R_1, \\ T &= S + G(N)(R_2 - R_1) & \text{if } R_2 \geqslant R_1, \end{aligned} \tag{1}$$

where $G(N)$ is the average first innings total number of runs in an N-over match (N is typically either 50 or 20). The target for team 2 is then the next integer above T.

2.1. The D/L model

To estimate the resources available to a team, the D/L method uses a model of the average runs remaining to be scored, Z. The D/L model for the average runs in the remaining u overs and with w wickets lost is given by

$$Z(u,w) = Z_0F(w)[1 - \exp\{-bu/F(w)\}], \tag{2}$$

where Z_0 is the asymptotic average runs with no wickets lost in an infinite number of overs. $F(w)$ is a positive decreasing step function with $F(0) = 1$ and is interpreted as the proportion of runs that are scored with w wickets lost compared with that of no wickets lost and, hypothetically, infinitely many overs available. That is, $F(w) = \lim_{u\to\infty} Z(u,w)/Z(u,0)$. The ratio

$$P_N(u,w) = Z(u,w)/Z(N,0) \tag{3}$$

gives the average proportion of runs still to be scored in an innings with u overs remaining and with w wickets lost, which Duckworth and Lewis present as the proportion of remaining resources. For brevity, we refer to this as remaining resources, although strictly speaking it is a proportion.

Duckworth and Lewis (2004) modified the original 1998 model for high scoring matches. The idea being that the resources remaining, for a given number of wickets lost, decrease linearly when a team is chasing a well above average target. In other words, each over has equal value and so the distribution of runs scored per over tends to be uniform, provided that the number of wickets lost remains the same. For this purpose they include an extra parameter which they call the match factor and is denoted by λ. In matches with well above average targets, the parameter λ scales down the rate parameter b. As a result Z tends to linearity with respect to u. The D/L upgraded model is given by

$$Z(u,w|\lambda) = Z_0F(w)\lambda^{n(w)+1}[1 - \exp\{-bu/\lambda^{n(w)}F(w)\}], \tag{4}$$

where $n(w)$ is a positive decreasing function with $n(0) = 5$. Strictly speaking, we should not be conditioning only on λ, but to distinguish Z in Eq. (4) from Z in Eq. (2) we follow this notation of Duckworth and Lewis and continue with it throughout the paper. In innings i ($i = 1, 2$), following n_i interruptions (the jth interruption stops play when u_{1j} overs remain and w_j wickets have been lost and play is resumed when u_{2j} overs remain), the resources available is given by

$$R_i = 1 - \sum_{j=1}^{n_i}(P_N(u_{1\,j}, w_j|\lambda) - P_N(u_{2\,j}, w_j|\lambda)). \tag{5}$$

2.2. Estimation of the D/L model

The parameters to be estimated are Z_0, b, $F(w)$, $n(w)$ and λ, and estimation can be done in two stages. In the first stage Z_0, b, and $F(w)$ are estimated from Eq. (2). In the second stage we first need a functional form for $n(w)$. Duckworth and Lewis (2004) does not reveal the functional form of $n(w)$. However, our experimentation with Tables 1 and 2 in Duckworth and Lewis (2004) revealed that $n(w) = \alpha + \beta F(w)$, where $\alpha = 2$ and $\beta = 3$. This functional form was confirmed to be that used in the D/L method in correspondence with Tony Lewis and Frank Duckworth. $F(w)$ is estimated non-parametrically with two constraints: $F(w) \geqslant F(w+1) > 0$ and $F(0) = 1$. Finally, λ is estimated on a match-by-match basis, after the first innings has been completed in a game.

The estimation methodology used by Duckworth and Lewis remains unknown, so we now describe the approach adopted here to fit both the D/L model and our modified D/L model. We first describe how we estimate Z_0, b and $F(w)$ and then describe estimation of λ.

2.2.1. Estimation of Z_0, b and $F(w)$

In correspondence with Tony Lewis and Frank Duckworth, it was revealed that $F(w)$ was first estimated from data, and then subjectively smoothed to produce more intuitive behaviour of $Z(u,w)$. Following advice from Duckworth and Lewis, we keep these values, as this is how the model is adopted in practice (these values of $F(w)$ are given later in Table 2 below).

Let $x_i(u,w)$ be the observed runs scored in the remaining u overs of the first innings of match i when w wickets have been lost. Similarly, let $\bar{x}(u,w)$ be the observed mean runs scored in the remaining first innings. First innings, and not second innings, data is used here (as in Duckworth and Lewis) because the scoring pattern in the second innings will be affected by the target set in the first