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Interfaces with Other Disciplines

Multistage multiproduct advertising budgeting

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ABSTRACT

We propose and analyze an effective model for the *Multistage* Multiproduct Advertising Budgeting problem. This model optimizes the advertising investment for several products, by considering cross elasticities, different sales drivers and the whole planning horizon. We derive a simple procedure to compute the optimal advertising budget and its optimal allocation. The model was tested to plan a realistic advertising campaign. We observed that the *multistage* approach may significantly increase the advertising profit, compared to the successive application of the *single stage* approach.

1. Introduction

In this article we address the Multistage Multiproduct Advertising Budgeting problem (for short, we will drop one M and call it the MAB problem). More specifically, by advertising budgeting we mean that we wish to decide the capital to be invested on advertising and how to allocate it in an optimal way. By multiproduct we mean that we simultaneously optimize the advertising campaigns of different products within the same company by using different media (television, radio, internet, etc.) and considering *cross product effects* [9]. By multistage, we mean that we optimize the advertising campaigns for the whole planning horizon.

Every year many companies spend thousands of euros to advertise and promote their products. An appropriate optimizing technology can help either to obtain better advertising results for a given budget or to reduce the advertising expenses. We are living the era of *Big Data*, where companies gather and manage huge data bases [13]. By using *market response models* we can transform this raw marketing information into 'ready to use' information [16]. For example, we can model the sales due to advertising as a function of the advertising investment. The direct use of these models is to study different outcomes to take a 'good' decision. A more effective use is to combine them with a utility function to construct an optimization model intended to give a 'best' decision. The relevance of the advertising budgeting problem for the marketing industry is discussed in [13].

The advertising budgeting problem has been addressed in literature from different perspectives. An introductory and interesting paper can be found in [9] where the authors propose a simple formula for calculating the optimal level of media spending in the case of a single product, single medium and a single stage. The *multiproduct* advertising budgeting problem is analyzed in [10], whereas the *multistage* advertising budgeting problem is studied in [26]. In some cases, due to the complexity of the formulation one has to use heuristic methods which produce good (suboptimal) solutions [29,22]. In other cases if the complexity of the formulation is moderate, depending on the focus of the model, one can use an optimal control approach [28,12,15] or a stochastic optimization approach [1,11,2], or a stochastic optimal control approach [5], or a game theory approach [21,23,8], or a goal programming approach [3], among others.

As we have mentioned, different aspects of the advertising budgeting problem have been considered. However, as far as we know, a multistage version of the multiproduct advertising budgeting problem has not yet been addressed in literature. Thus, for example, [26] considers a multistage setting but for a single product whereas, [10] considers the multiproduct case but for a single stage. However, as we will illustrate in our case study, relevant savings can be achieved if the planning stages are optimized simultaneously (multistage models) compared to the optimization stage-by-stage (single stage models). Therefore, the main contribution of this paper is to propose and analyze a *multistage version* of the multiproduct advertising budgeting problem.

The objective of this paper is to propose and analyze an effective formulation for the MAB problem. We are interested in calculating the optimal advertising budget and its optimal allocation. We will

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try to answer the following questions: (a) Which is the optimal multiproduct advertising budget for the whole planning horizon? (b) Given an advertising budget, how can we optimally allocate it along the planning horizon? (c) Is it important to consider multistage models? or on the contrary, is it enough to consider single stage models?

We are also concerned with effectiveness. In [18] it is pointed out that a model that is to be used by a manager should be simple, robust, easy to control, adaptive, as complete as possible and easy to communicate with. In line with this recommendation, the MAB model that we propose is simple but realistic enough to be used in the advertising industry. Furthermore, from a mathematical point of view, it corresponds to a concave maximization problem which is numerically tractable and allows for the computing of a (global) optimal solution with moderate computational effort.

The remainder of the paper is organized as follows: In Section 2 we will formulate the MAB problem (unconstrained and constrained case). In Section 3 we will derive a simple procedure to compute the optimal advertising budget and its optimal allocation. In Section 4 a realistic case study will allow us to illustrate the effectiveness of the model as well as the theoretical concepts of the third section. In Section 5 we will give some conclusions. Appendix A contains the proofs of all the theoretical results of this article. Appendix B contains the data for the case study in Section 4.

2. Problem formulation

In order to formulate the Multistage Multiproduct Advertising Budgeting (MAB) problem, we distinguish between baseline sales (sales that one would expect without advertising) and *sales due to advertising*. The objective of the MAB model is to maximize the profit of the sales due to advertising. Expressed in a different way, we will consider the profit of the sales due to advertising as the measure of the advertising effectiveness. Prior to formulate the MAB problem, we briefly review some key concepts in the advertising industry (see, for example, [16] for more details). *Reach* is the proportion of the target audience exposed to at least one insertion of the advertisement [9]. We call this proportion the reach audience. *Frequency* is the average number of times a person from the reach audience is exposed to an advertisement. *Exposure* to an advertisement involves reach and frequency and can be measured in Gross Rating Points: GRPs $\equiv$ reach $\times$ frequency. For example, a purchase of 100 GRPs could mean that 100% of the market is exposed once to an advertisement or that 50% of the market is exposed twice [16]. According to [2], advertising is measured in GRPs and not in euros, since there are two advantages of using GRPs. First, GRPs provide a more accurate picture of advertising input than advertising expenditures since it is not clear how much advertising exposure can be purchased for a given budget. Second, most media buying is done in terms of GRPs and managers evaluate the effectiveness of their campaigns in terms of demand generated per GRP.

The impact of advertising effort spreads over time and the advertising effort in one stage is cumulated with past advertising efforts. In this respect, we will use the variable so called *adstock* [4,16], which is a measure of the past and current advertising effort that is effective in the current stage. For brand management, market response models provide a basis for fine tuning marketing mix variables (*marketing mix* for short), such as price, sales promotions, advertising copy, media selection, timing, and other brand-specific marketing factors. Furthermore, the marketing mix has to take into account the *market segmentation,* that is, the distinct consumer groups, each one characterized by the same needs and behaviors [7,6]. The largest category of empirical response models are those dealing with sales and market share as dependent variables. Companies want to know what influences their sales (the *sales drivers* or, for short, *drivers*). They want to know how to set the marketing mix so that they can control their sales. One of the limitations of the MAB model that we present is that it does not take the product price as a sales driver, that is, as a decision variable (prices are input data). All the other above mentioned drivers (sales promotions, advertising copy, media selection and timing) can be taken into account in our MAB model. Furthermore, by an abuse of terminology, when we talk about sales due to advertising, we refer to sales due to these drivers (analogously with advertising budgeting, advertising allocation, etc.).

2.1. Notation

In our formulation of the MAB problem we consider:

Indexes:		
t	Index for stages, $t \in \mathcal{T} = \{1,\ldots,T\}$	
i	Index for products, $i \in \mathcal{I} = \{1,\ldots,I\}$	
j	(Auxiliary) index for products, $j \in \mathcal{I}$	
k	Index for sales drivers for product i, $k \in \mathcal{K}_i = \{1,\ldots,K_i\}$	$i \in \mathcal{I}$
$\mathcal{TIK}_i$	Stands for $\mathcal{T} \times \mathcal{I} \times \mathcal{K}_i$	$i \in \mathcal{I}$
Parameters:		
a_{tijk}	Sales of product i in stage t induced by one unit of driver jk where $j \neq i$ (note that, for simplicity, to refer to the kth driver of product j, we use the expression 'driver jk'), $a_{tijk} \in \mathbb{R}$	$tijk \in \mathcal{TIIK}_i$
c_{tik}	Cost of driver ik in stage $t \in \{1,\ldots,T+1\}$, $c_{tik} > 0$	$ik \in \mathcal{IK}_i$
δ_{ik}	Retention rate of the advertising effort from stage to stage for driver ik, $\delta_{ik} \in]0,1[$	$ik \in \mathcal{IK}_i$
p_{ti}	Profit per unit of product i in stage t, $p_{ti} > 0$	$ti \in \mathcal{TI}$
$\tilde{x}_{0ik}$	Advertising effort of driver ik previous to the first stage, $\tilde{x}_{0ik} \geqslant 0$	$ik \in \mathcal{IK}_i$
Functions:		
R_{tik}	Sales of product i in stage t due to driver ik	$tik \in \mathcal{TIK}_i$
L_{tijk}	Sales of product i in stage t due to driver jk where $j \neq i$ ('cross product effect')	$tijk \in \mathcal{TIIK}_j$
S_{ti}	Sales of product i in stage t due to advertising	$ti \in \mathcal{TI}$
P	Profit function (information aggregated by products)	
$\widetilde{Q}$	Profit function (information aggregated by drivers)	
Variables:		
g_{tik}	Investment in GRPs of driver ik in stage t	$tik \in \mathcal{TIK}_i$
x_{tik}	Cumulated advertising effort of driver ik in stage t ('adstock')	$tik \in \mathcal{TIK}_i$

2.2. The multiproduct sales response function

In the MAB problem one can maximize different utility functions. One common approach is to maximize the advertising profitability [9]:

$$\text{Advertising profitability} = \text{Profit} \times \text{Sales}_g - \text{Cost}_g$$

where 'Profit' is the profit per unit, 'Sales$_g$' corresponds to the sales due to advertising which is a function of g, the number of GRPs,