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Discrete Optimization

The accessibility arc upgrading problem

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ABSTRACT

The accessibility arc upgrading problem (AAUP) is a network upgrading problem that arises in real-life decision processes such as rural network planning. In this paper, we propose a linear integer programming formulation and two solution approaches for this problem. The first approach is based on the knapsack problem and uses the knowledge gathered from an analytical study of some special cases of the AAUP. The second approach is a variable neighbourhood search with strategic oscillation. The excellent performance of both approaches is demonstrated using a large set of randomly generated instances. Finally, we stress the importance of a proper allocation of scarce resources in accessibility improvement.

1. Introduction

Accessibility is formally defined by Donnges (2003) as the degree of difficulty people or communities have in accessing locations for satisfying their basic social and economic needs. This concept has been recognised to play an important role in the quality of life as well as the potential for development of communities and regions. The road network is one of the main elements that contributes to the accessibility. This is particularly true in rural areas of lesser-developed countries, where the road network ensures the accessibility to the economic and social infrastructure and to facilities, such as hospitals, usually located in regional centres or in more developed cities. In this paper, we study the accessibility arc upgrading problem (AAUP), a network upgrading problem in which resources have to be allocated in order to improve the accessibility to a set of vertices in a network. In the domain of rural road network planning, this problem arises when allocating resources to upgrade roads of a rural transport network, in order to improve the access that communities in small villages have to regional centres. We proceed by giving a precise description of this problem.

The AAUP can be described as follows: Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a directed connected graph in which the vertex set $\mathcal{V}$ is partitioned into two different sets $\mathcal{V}_1$ and $\mathcal{V}_2$. Vertices in $\mathcal{V}_1$ are called centres, while vertices in $\mathcal{V}_2$ are called regular vertices. Each arc e in $\mathcal{E}$ has a current level and a set of possible upgrading levels. The level of an arc determines the time required to traverse it. An upgrading cost is incurred when improving an arc from its current level to a specific upgrading level. There is a total budget B to upgrade the level of some arcs. For each vertex j in $\mathcal{V}_2$, a weight w_j (e.g., number of inhabitants) is given. We define as measure of the accessibility of regular vertex j the travel time from j to the closest centre i in $\mathcal{V}_1$. An *upgrading strategy* specifies a set of arcs to be upgraded and the level to which each of them has to be improved. The objective is to find an upgrading strategy that does not exceed the budget B and minimises the weighted sum of the accessibility measures, i.e., the weighted sum of the times required to travel from each vertex j in $\mathcal{V}_2$ to its nearest centre i in $\mathcal{V}_1$.

The rest of this paper is structured as follows. In Section 2, we propose a linear integer programming formulation of the AAUP problem. Section 3 reviews the literature, and, in Section 4, we analyse special cases. Section 5 proposes heuristic methods for the AAUP, and, in Section 6, we test these methods on randomly generated instances. Section 7 discusses the potential practical impact of the AAUP. Finally, Section 8 summarises the main contributions of this work and highlights some opportunities for future research on this topic.

2. Mathematical formulation

Based on the mathematical formulation described in Campbell and Lowe (2006), the AAUP can be formulated as a non-linear binary programming model, as shown by Maya Duque and Sörensen (2011). In this paper, we propose an alternative formulation in which the AAUP is defined as a special case of a more general problem called budget constrained minimum cost flow problem (BC-MCFP).

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In the BC-MCFP, a given amount of flow has to be sent from a set of supply vertices or sources, through the arcs of a network, to a set of demand vertices or sinks. For each existing arc in the network, there is set of possible upgrading levels. Therefore, for each existing arc, we define one new arc per possible upgrading level connecting the same pair of vertices. Thus, in the BC-MCFP formulation, $\mathcal{E}$ represents the augmented set of arcs that contains all the original arcs and the arcs generated for each possible upgrading level. For each arc in $\mathcal{E}$, there is a cost per unit of flow, and a fixed cost associated with the use of the arc. In our particular setting of the BC-MCFP, there is no fixed cost for using an arc at its lowest level, but that cost increases with the upgrading level. The cost per unit of flow decreases as the arc is upgraded. The problem is to find a minimum cost flow, such that the sum of the fixed costs incurred by using some of the arcs at an upgraded level is limited to a fixed budget. Basically, this problem is a minimum cost flow problem that involves an additional set of decision variables related to the upgrading decisions.

Consider the variable x_e which is equal to the flow over arc e, and a binary variable y_e which is equal to 1 if the arc e is used, and 0 otherwise. Let $\delta^+(i)$ and $\delta^-(i)$ be the forward and backward stars of vertex i, respectively. Furthermore, let parameter d_i denote the demand or supply in vertex i, and let p_e and c_e represent the fixed cost of using arc e, and the cost per unit of flow over arc e, respectively. Note that d_i is positive for supply vertices and negative for demand vertices. A formulation for the BC-MCFP is as follows:

$$\min \sum_{e \in \mathcal{E}} c_e x_e \qquad (1)$$

s.t.

$$\sum_{e \in \delta^+(i)} x_e - \sum_{e \in \delta^-(i)} x_e = d_i \qquad \forall i \in \mathcal{V} \qquad (2)$$

$$x_e \leqslant M y_e \qquad \forall e \in \mathcal{E} \qquad (3)$$

$$\sum_{e \in \mathcal{E}} p_e y_e \leqslant B \qquad (4)$$

$$\sum_{e:e=(i,j)} y_e \leqslant 1 \qquad \forall i,j \in \mathcal{V} : (i,j) \in \mathcal{E} \qquad (5)$$

$$0 \leqslant x_e \leqslant a_e \qquad \forall e \in \mathcal{E} \qquad (6)$$

$$y_e \in \{0,1\} \qquad \forall e \in \mathcal{E} \qquad (7)$$

The objective function (1) minimises the total flow cost. The constraints in (2) ensure that the demand for each sink vertex j is satisfied and that the supply of each source i is not exceeded. The constraints in (3), where M denotes a large number, enforce that flow can only pass through arcs that have been selected for use. Constraint (4) imposes an upper bound B on the total upgrading cost. The constraints in (5) ensure that at most one arc connecting each pair of vertices is chosen. Note that these constraints are not needed when the arcs are uncapacitated. Finally, constraints (6) and (7) define the type and the bounds for the decision variables. In constraints (6), a_e represents the capacity of arc e.

We now show that the AAUP is a special case of the BC-MCFP. Consider an instance of the AAUP as described in Section 1. Each regular vertex acts as a sink, while each centre is a supply vertex. The value of d_j for a regular vertex j is set to $-w_j$, while the value of d_i for each centre i is set to the total demand on the network (i.e., the sum of the w_j values for all j in $\mathcal{V}_2$). Then, we create one dummy demand vertex connected to each of the centres. The fixed cost and cost per unit of flow for the arcs connecting the dummy vertex and the centres are set to 0, while the d_i value of the dummy vertex is set to $-(|\mathcal{V}_1| - 1)\sum_{j \in \mathcal{V}_2} w_j$. Solving the resulting instance of the BC-MCFP yields a solution for the corresponding instance of the AAUP. Fig. 1 shows the transformation of an AAUP into a BC-MCFP, schematically.

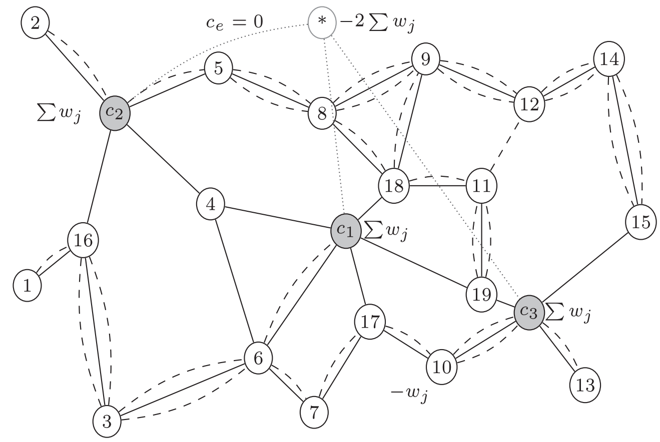

Fig. 1. Transformation of the AAUP into a BC-MCFP.

In Fig. 1, the vertices c_1, c_2 and c_3 are the centres, while the vertices 1–19 represent the regular vertices. The solid lines correspond to existing arcs of the network, while the dashed lines are possible upgrading levels of the existing arcs. The vertex labelled with an asterisk represents the dummy demand vertex and the grey dotted lines are the arcs that connect the dummy vertex to the centres.

3. Literature review

In this section, we review the literature that is relevant for the AAUP. We first concentrate on the network upgrading problem. Afterwards, we extend the review to consider the accessibility factor within the upgrading network problem.

Although several authors have addressed network upgrading problems, the literature is not as extensive as it is for other problems within the domain of network design. Krumke et al. (1998) distinguish two kinds of upgrading problems depending on whether the focus is on upgrading the arcs or upgrading the vertices. The authors propose a bi-objective approach for both types of problems. In that approach, a sub-class of graph $\mathcal{S}$ is considered (e.g., the set of spanning trees) and a budget or target value is defined for the first objective. The goal is to find a network within the fixed budget that belongs to $\mathcal{S}$ and minimises the second objective. Results on the complexity of a number of node-based and edge-based upgrading problems are presented. In particular, the case in which the objectives are defined as minimising the cost of improving the network and minimising the total length of the minimum spanning tree is shown to be NP-hard for trees and general networks. Drangmeister et al. (1998) study a related problem that looks for an optimal reduction strategy (i.e., shortening some of the edges) such that a budget constraint is satisfied and the total length of a minimum spanning tree in the modified network is minimised. Some NP-hardness results, even for simple classes of graphs, are presented, as well as some approximation algorithms.

Campbell and Lowe (2006) address two q-upgrading arc problems that involve finding the best q arcs to upgrade in a network. The q-upgrading arc diameter problem requires finding q arcs to upgrade such that the travel time on the maximum shortest path between any origin–destination pair (i.e., the diameter of the network) is minimised. The q-upgrading arc radius problem requires finding q arcs to upgrade and locating the vertex centre, i.e., the node for which the maximum shortest path to the other nodes in the network (i.e., the radius of the network) is minimised. The two problems are shown to be NP-hard on general graphs, but polynomially solvable on trees. A variant of the problems, which involves a budget constraint, is also studied. It is shown that these