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Concepts for safety stock determination under stochastic demand and different types of random production yield

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ABSTRACT

We consider a manufacturer's stochastic production/inventory problem under periodic review and present methods for safety stock determination to cope with uncertainties that are caused by stochastic demand and different types of yield randomness. Following well-proven inventory control concepts for this problem type, we focus on a critical stock policy with a linear order release rule. A central parameter of this type of policy is given by the safety stock value. When non-zero manufacturing lead times are taken into account in the random yield context, it turns out that safety stocks have to be determined that vary from period to period. We present a simple approach for calculating these dynamic safety stocks for different yield models. Additionally, we suggest approaches for determining appropriate static safety stocks that are easier to apply in practice. In a simulation study we investigate the performance of the proposed safety stock variants.

1. Introduction

In environments where not only customer demand is stochastic but also production is exposed to random yields, inventory control becomes an extremely challenging task. Yield uncertainties frequently occur in the agricultural sector or in the chemical, electronic and mechanical manufacturing industries (see Gurnani et al., 2000; Jones et al., 2001; Kazaz, 2004). Here, random supply can appear due to different reasons such as weather conditions, production process risks or imperfect input material. Semiconductor manufacturing in the electronic goods industry is an especially striking example where high yield losses of about 80% on average are met (see Nahmias, 2009, p. 392). More recently, yield problems gained also relevance in the remanufacturing industry where the output of disassembly operations often is highly uncertain because of limited knowledge of the condition of used products (see Ilgin and Gupta, 2010; Panagiotidou et al., in press).

The main challenge for production and inventory planning in this context is that yield losses often are hard to predict so that their variances are too high to be ignored. To cope with the influence of risks that concern demand and yield variability, two control parameters can be used in an MRP-type production control system: a safety stock and a yield inflation factor that accounts for yield losses (see Inderfurth, 2009; Nahmias, 2009, p. 392; Vollmann et al., 2005, p. 485). In general, it is not necessary to implement safety stocks for all items of a multi-level MRP-system since a safety stock for the final product automatically increases the requirements for products on the lower stages (see Nahmias, 2009, p. 388). However, for items with significantly variable yield it is strongly recommended to install safety stocks (see Silver et al., 1998, p. 613).

Scientific contributions which deal with theory-based determination of control parameters originate from research in the field of stochastic inventory control problems. In previous articles on optimal inventory control under stochastic yields, considering a single-item inventory problem under periodic review, several authors (see Gerchak et al., 1988; Henig and Gerchak, 1990) have analyzed that the optimal policy for cost minimization results in a critical stock (*CS*) rule in combination with a non-linear order release function. This rule prescribes that an order is only released in a specific period if the inventory level does not exceed *CS*. If the inventory is lower, the order quantity to be released is increasing with the size of the *CS* undershoot. The order quantity, however, is not proportional to the difference between *CS* and current stock level. This type of order release function, however, is cumbersome to calculate and difficult to apply in practice. In contrast, the way how demand and yield risks are handled in practical MRP systems results in applying a *CS*-rule with a linear order release function where the *CS* is composed of a safety stock and the expected demand during lead time and control period (see Inderfurth, 2009). So it is quite evident that several attempts have been made to develop powerful linear approximations to the non-linear order rule.

Approaches by Henig and Gerchak (1990) and Zipkin (2000) which solely focused on coping with yield randomness by

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appropriately inflating the order size did not turn out to be sufficiently successful. In contrast, procedures that tried to incorporate the yield risk in an appropriate manner in the *CS* determination for a linear rule showed a much better performance. In this context, the best performance is reached by a procedure developed by Bollapragada and Morton (1999) who present an advanced linear heuristic for the multi-period case under zero production lead time and linear costs for production, stock-keeping, and backlogging. Following this approach the *CS* is calculated as a simple closed-form expression from an extended newsvendor analysis where the yield risk is also taken into account. The expected yield loss is incorporated by inflating the stock deviation from *CS* by a yield inflation factor (denoted by *YIF*) which is simply chosen as the reciprocal of the mean yield rate. In a numerical study, using a dynamic programming procedure, Bollapragada and Morton compare the results of the linear heuristic with the optimal non-linear order release rule and show that in terms of cost deviation their heuristic performs very well in most instances. Inderfurth and Transchel (2007) detect an error in the analytic procedure of Bollapragada and Morton that is responsible for a steady deterioration of their heuristic for parameter constellations which correspond to increasing service levels. In a recent study, Huh and Nagarajan (2010) revisit the linear control rule problem with zero lead time as addressed by Bollapragada and Morton and develop a numerical approach for calculating optimal values of *CS* for a given *YIF*. They prove that for any given *YIF* the average costs are convex in *CS* and exploit this property in deriving a fairly simple calculation procedure. They also compare the performance of different methods for determining the *YIF* suggested in literature by a comprehensive simulation study.

The only approaches for developing linear control rules with non-zero lead times are found in Inderfurth and Gotzel (2004) and Inderfurth (2009). Using a fixed *YIF*, but a time-dependent (i.e. dynamic) *CS*, this work extends the parameter determination approach in Bollapragada and Morton to cases with arbitrary lead times. The main idea is to determine appropriate safety stocks as parameter for the linear control rule that enable a quite good approximation to the non-linear order release function, even in cases with positive lead time and outstanding past orders which generate an additional yield risk. This stream of research is further extended in this paper as the multiple-period lead time case is regularly met in practice, particularly in an MRP environment. One extension refers to the development and performance analysis of static instead of dynamic safety stocks in a multi-period lead time environment. As a second extension additional types of yield randomness are incorporated in the analysis of control rule parameters. This is because up to now all contributions only referred to production environments where the process yield is stochastically proportional to the production input quantity.

In our study we consider both arbitrary lead times and two additional well-known types of yield randomness (see Yano and Lee, 1995), namely binomial and interrupted geometric yield. The yield models under consideration mainly differ in the level of correlation existing for individual unit yields within a single production batch. We show how for different models safety stocks can easily be determined following the same theoretical concept when using a linear order release rule with a *YIF* that is the reciprocal of the mean yield rate. We show that in case of non-zero lead time even under stationary conditions it is straight-forward that safety stocks will vary from period to period. In order to facilitate applicability of safety stock usage and facilitate smoothed production orders, we additionally present alternative approaches of how these dynamic safety stocks can be transformed into static ones.

The rest of our paper is organized as follows. In Section 2 the linear control rule applied for order release is formalized and the role of safety stock within this rule is clarified. Section 3 introduces different models of stochastic yield processes, while Section 4 presents closed-form safety stock expressions for each type of random yield. In Section 5 the different safety stock formulas for each considered type of yield randomness are compared and evaluated with respect to their relative performance. Section 6 concludes this contribution with some managerial insights and gives a brief outlook on future research.

2. Linear control rule

In this section we present details of the linear control rule which is used to cope with demand and yield risks in the multi-period infinite-horizon case and with arbitrary lead times. Like in Bollapragada and Morton (1999) and in Huh and Nagarajan (2010), the basic idea is that this rule is applied in the context of a problem environment with stationary stochastic demand and yield processes in order to minimize the long-run expected holding and backlogging costs. Following a linear order release rule under arbitrary lead times means that in each period the current net inventory and open orders are combined to an appropriate inventory position which is compared to a critical stock level. In the case of a stock level undershoot, this difference is magnified by a yield inflation factor and, in this enlarged size, represents the next order quantity.

For the formal description of this linear control rule under general types of a stochastic production yield the following notation is used:

Q_t	released order quantity in period t
CS_t	critical stock for period t (potentially time-dependent)
x_t	inventory position, expected in period t
SST_t	safety stock for period t (potentially time-dependent)
YIF	yield inflation factor
λ	production lead time
$\widetilde{Y}(Q)$	random yield (number of good units from a production batch size Q)
$\overline{Y}(Q)$	expected yield $(= E[\widetilde{Y}(Q)])$
$\widetilde{Z}$	random yield rate, defined as $\widetilde{Y}(Q)/Q$
$\widetilde{D}_t$	i.i.d. random demand in period t with expectation μ_D and variance σ_D^2
α	critical cost ratio (depending on holding and backlogging cost)

Following a critical stock rule with a linear order release mechanism characterized by the parameters CS_t and *YIF*, an order Q_t in period t depends on the expected inventory position x_t in the following way:

$$Q_t(x_t) = \max\{(CS_t - x_t) \cdot YIF; 0\} \tag{1}$$

Like in Bollapragada and Morton (1999), we fix the yield inflation factor, *YIF*, in such a way that it just compensates for the expected yield loss of an order and does not depend on the variance of the yield process. Like demand risk, this yield variability is completely coped with by the critical stock, CS_t. Different from standard approaches, parameter CS_t is allowed to be time-dependent in a stationary model environment because under lead times of multiple periods it has to reflect that the current yield risk from open production orders can change from period to period along with the distribution and size of the past orders. In this context, it has to be noted that under stochastic yield the inventory position cannot be defined in the traditional way as net inventory plus sum of outstanding orders, since the in-transit inventory from these orders is random. In order to maintain the traditional approach, it is straightforward to replace the random yield by the