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Geometrically nonlinear free vibration analysis of axially functionally graded taper beams

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ABSTRACT

Large amplitude free vibration analysis is carried out on axially functionally graded (AFG) tapered slender beams under different boundary conditions. The problem is addressed in two parts. First the static problem corresponding to a uniform transverse loading is solved through an iterative scheme using a relaxation parameter and later on the subsequent dynamic problem is solved as a standard eigenvalue problem on the basis of known static displacement field. The mathematical formulation of the static problem is based on the principle of minimum total potential energy, whereas Hamilton's principle has been applied for the dynamic analysis. To account for the geometric non-linearity arising due to large deflection, nonlinear strain displacement relations are considered. The dynamic behaviour has been presented in the form of backbone curves in a dimensionless frequency–amplitude plane. The results are successfully validated with the previously published results.

1. Introduction

Non-uniform beams with variable cross-section provide a suitable distribution of mass and strength for engineering structures. These structural elements are commonly used in various engineering applications, such as, gas turbines, wind turbines, helicopter rotor blades, ship propellers, robot arms, space and marine structures etc. [12]. Their wide-spread usage in various advanced branches of civil, mechanical and construction industries is due to their ability to cater to different structural requirements. Hence, prediction and determination of dynamic behaviour of these components have been an area of great interest among researchers.

Functionally graded materials (FGMs) are new and advanced class of inhomogeneous composites, which are obtained by combination of two or more constituent materials, mixed continuously and functionally according to a given volume fraction. As a result, material properties become a function of spatial position and a continuous variation from one surface to another can be achieved. In this respect, FGMs are advantageous over contemporary laminated composites as property variation is continuous and thus eliminate stress concentration [30]. Whereas, laminated composites suffer from the disadvantage of discontinuity at the layer interface and subsequent stress concentration. In the modern context, FGMs find extensive application in aerospace, civil and mechanical engineering fields [43], especially, where, unevenly distributed thermal, chemical or mechanical loads are present.

The variation of material properties in functionally graded (FG) beams may be oriented in transverse (thickness) direction or longitudinal/axial (length) direction or both. An exhaustive literature review of the relevant domain reveals that majority of the studies are concentrated on free vibration analysis of FG beams with material property variation along the depth of the beam. In case of a free vibration study of a structure the main objective is to determine the natural frequencies corresponding to various modes of vibration of the system. Several different techniques and methodologies have been adopted for this purpose by different researchers [6,44] derived the governing equations using Hamilton's principle while employing different higher order shear deformation theories and obtained the solution to these equations using Navier solution method. Analysis of free vibration of FG beams was also carried out by [23,41,42]; who used different techniques to solve the governing equations obtained from application of Hamilton's principle. Nguyen et al. [30] introduced a method involving first-order shear deformation beam theory where the improved shear stiffness matrix was derived from the in-plane stress and equilibrium equation.

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Nomenclature

A_0	cross-sectional area of the beam at the root
b	width of the beam
c_i	unknown coefficients for static analysis
d_i	unknown coefficients for dynamic analysis
E_0	elastic modulus of the beam material at the root
$\{f\}$	load vector
I_0	moment of inertia of the beam at the root
$[K]$	stiffness matrix
$[K_s]$	static stiffness matrix
L	length of the beam
$[M]$	mass matrix
nw, nu	number of constituent functions for w and u respectively
ng	number of Gauss points
p	magnitude of uniformly distributed load
t_0	thickness of the beam at the root
T	kinetic energy of the system
u	displacement field in x-axis
U	strain energy stored in the system
V	potential energy of the external forces
w	displacement field in z-axis
w_{max}	maximum deflection of the beam
α	taper parameter
δ	variational operator
ε_x^b, ε_x^s	axial strains due to bending and stretching respectively
ρ_0	density of the beam material at the root
τ	time coordinate
ω_1	first natural frequency
ω_{nl}	nonlinear frequency parameters
ξ	normalized axial coordinate
π	total potential energy of the system
ψ_i	set of orthogonal functions for u
ϕ_i	set of orthogonal functions for w

It is found that finite element techniques are quite popular in analyzing the dynamics of FG beams. Chakraborty et al. [7] developed a new beam element also based on the first order shear deformation theory to study the free vibration of FG beams. Hemmatnezhad et al. [15] investigated the nonlinear behavior of FG beams using finite element formulation. Von Karman type nonlinear equations along with Timoshenko beam theory were used for the analysis. Piovan and Sampaio [32] studied the free vibration of axially moving thin-walled beam with annular cross-section using finite element method. Ke et al. [21] studied the nonlinear free vibration of FG beams using Galerkin's method. The nonlinear equations were based on Von Karman geometric nonlinearity and the governing equations were solved using direct numerical integration method and Runge-Kutta method.

The Ritz method along with an improved third order shear deformation theory was used by Ref. [45]. Li [24] adopted a new unified approach where a single fourth-order governing partial differential equation was derived. The analysis by [33] was based on classical and first order shear deformation theories where the governing equations were obtained using Rayliegh-Ritz method. Simsek [38] dealt with the classical, first and higher shear deformation theories and derived the equations of motion employing Lagrange's equations. Giunta et al. [11] worked with several axiomatic refined theories and derived the governing differential equations by variationally imposing the equilibrium through the principle of virtual displacements. Murin et al. [29] derived fourth-order differential equation for the FG beam and used linear beam theory to establish equilibrium and kinetic beam equations. Simsek and Kocaturk [39] used Lagrange's equations along with Euler–Bernoulli beam theory to study the free vibration behavior of FG beams under the action of concentrated moving loads. A total Lagrangian formulation was used by [1] to investigate the effects of geometric nonlinearity on the static and dynamic response of FG beams. Lu and Chen [26] obtained semi-analytical solutions for the free vibration of orthotropic FG beams using a hybrid state-space differential quadrature method along with an approximate laminate model. Kapuria et al. [20] presented a theoretical model and its experimental validation for the free vibration of a layered FG beam. Some research works are also available on the effect of nonlinear elastic foundations on free vibration behavior of FG beams [10,19,31,47]. The governing equations were based on Euler-Bernoulli beam theory and solved using Galerkin's method and He's variational iteration method.

A few researchers have concentrated on the free vibration of FG beams where the material property variation is along the length of the beam. Simsek et al. [40] derived the equation of motion by using Lagrange's equations and Newmark method was employed to find the dynamic responses of AFG beam. Shahba et al. [35–37] and Shahba and Rajasekaran [34] studied the free vibration and stability analysis of Euler-Bernoulli and Timoshenko beams through finite element approach and various numerical analysis methods. Alshorbagy et al. [4] employed numerical FEM and Euler-Bernoulli beam theory to investigate the dynamic characteristics of FG beams. Huang et al. [18] presented a new approach for investigating the vibration behaviors of non-uniform AFG Timoshenko beams by changing the coupled governing equations to a single governing equation by introducing an auxiliary function. Huang and Li [16,17] studied the dynamic and buckling behavior of AFG tapered beams by reducing the corresponding governing differential equation to Fredholm integral equations. Aydogdu [5], Elishakoff et al. [9] and Wu et al. [46] investigated the free vibrations of AFG tapered beams using the semi inverse method. Mazzei and Scott [27] studied stability and vibration of AFG tapered shafts loaded by axial compressive forces. Li et al. [25] derived the characteristic equations in closed form for exponentially graded beams with various boundary conditions. Kein [22] investigated the large displacement response of tapered AFG cantilever beams by finite element method. Hein and Feklistova [14] studied the vibrations of non-uniform FG beams with various boundary conditions using the Euler–Bernoulli theory and Haar wavelets. Akgoz and Civalek [3] performed vibration response analysis of AFG tapered micro beams with Euler-Bernoulli beam theory and modified couple stress theory, by utilizing Rayleigh–Ritz solution method. The authors [2] also investigated buckling problem of linearly tapered cantilever micro-columns of rectangular and circular cross-section on the basis of modified strain gradient elasticity theory.

Literature review reveals that a substantial amount of research work is focused on the field of free vibration study of depth-wise functionally graded beams, while relatively fewer research studies are available for AFG beams. Works on large amplitude free vibration, specifically variation of loaded natural frequencies with external transverse loading of AFG taper beams is limited. It should be pointed out that a vast majority of research papers deal with a particular type (Linear) of taper profile, while the emphasis remains on developing new methods to determine the natural frequencies of the system. Hence, the present study is taken up with