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Optimal design of FIR high pass filter based on L_1 error approximation using real coded genetic algorithm

Apoorva Aggarwal, Tarun Kumar Rawat, Manjeet Kumar*, D.K. Upadhyay

Department of Electronics & Communication, Netaji Subhas Institute of Technology, Delhi, India

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ABSTRACT

In this paper, an optimal design of linear phase digital finite impulse response (FIR) highpass (HP) filter using the L_1-norm based real-coded genetic algorithm (RCGA) is investigated. A novel fitness function based on L_1 norm is adopted to enhance the design accuracy. Optimized filter coefficients are obtained by defining the filter objective function in L_1 sense using RCGA. Simulation analysis unveils that the performance of the RCGA adopting this fitness function is better in terms of signal attenuation ability of the filter, flatter passband and the convergence rate. Observations are made on the percentage improvement of this algorithm over the gradient-based L_1 optimization approach on various factors by a large amount. It is concluded that RCGA leads to the best solution under specified parameters for the FIR filter design on account of slight unnoticeable higher transition width.

1. Introduction

Digital filters are frequency selective device, which convolves the discrete signal amplitude with the specified impulse response in frequency domain. Thus, the filter extracts useful part of the input signal lying within its operating frequency range. Two broad categories in which digital filters are classified based on different criteria are finite impulse response (FIR) filter and infinite impulse response (IIR) filter [1]. The output of FIR filter depends on present and past values of input, hence there is no feedback network and are realized non-recursively. On the other hand, the output of IIR filter depends not only on previous inputs, but also on previous outputs with theoretically infinite impulse response in time and requires more storage element for the recursive IIR filter. FIR filter approaches the ideal response with the increase in filter order, thus the complexity and processing time increases. Whereas, IIR filters tends to be ideal at lower filter order on the account of obtaining non-linearity in phase and instability issues. In digital filtering applications, the FIR filters are often preferred over the IIR because of their inherent stability and the ability to provide a linear phase response over a wide frequency range.

The problem of filter design can be viewed as a constraint minimization problem, to meet all the requirement with an acceptable degree of accuracy for an optimal design. To find more efficient techniques and application based optimal solution is still an active field of research for the research community. There are different established techniques that exist for the design of FIR filter and its implementation [1,2]. The least-squares (LS) method minimizes the mean squared error (with the L_1-norm based fitness function) and is solved using the normal equations by Gaussian elimination. LS filters are popular and are extensively used in many applications [1,3–6]. Minimizing the LS error has the physical interpretation of energy minimization, which is also related to the signal to noise ratio colligated with the signals to be filtered. The resulting optimal filter demands the solution of a single linear system of equations, which can be solved efficiently. The eigenfilter method is one of the fastest ways to obtain an approximate filter [5]. This algorithm computes an eigenvector of an appropriate matrix to obtain the optimal filter coefficients in the LS sense but requires a large amount of calculations for solving the eigenvalue problem.

LS filters, however, results in overshoot at the discontinuity. Thus, the minimax measure of error is computed which minimizes the maximum absolute error value (obtained by varying the filter coefficients) in the filter response [7,8]. FIR filter design is achieved

* Corresponding author. Tel.:+91 8860010669.
E-mail addresses: 16.apoorva@gmail.com (A. Aggarwal), tarundsp@gmail.com (T.K. Rawat), manjeetchhillar@gmail.com (M. Kumar), upadhyay_d@rediffmail.com (D.K. Upadhyay).

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on minimization of Chebyshev error (with the L_∞-norm based fitness function) using linear programming techniques [9,10]. It is more efficiently accomplished with the Parks−McClellan algorithm [1,7] which renders a minimum Chebyshev error by employing the Remez exchange algorithm obtaining equal ripples in frequency domain.

Another common method, related to the least-squares approach, is the windowing technique with easy implementation [11,12]. In this, LS error approximation to an ideal low-pass filter is truncated by the multiplication of the infinite ideal impulse response and a relatively simple time domain window. They find importance in short-time Fourier analysis and their use in filter design evolved from the demand of having a simple method which reduces the Chebyshev error resulting from the Gibbs phenomenon in the LS error approximation of an ideal frequency response that has a discontinuity. However, it destroys the minimum squared error optimality of the original approximation and have inexplicit effects on the frequency response. Choice has to be made from a variety of window functions on the grounds of the amount of reduction of the ripples, acceptable range of the transition region and on the ease of calculations.

From last few decades, numerous significant results and effective algorithms have been developed in the L_1-approximation theory [13]. Conventionally, L_1-norm was adopted in several engineering applications, particularly in robust estimation problems, basis pursuit and sparse representations [14]. In the design of filters, the study of L_1-approximation is mainly concerned with the problems of uniqueness and characterization [15] and with the purposes of smoothing and deconvolution [16,17]. An L_1-approximation based method for the synthesis of digital FIR filters with the objective to optimize the filter parameters such that their frequency responses approaches to that of ideal ones was introduced in [18]. This was achieved by minimizing the L_1-norm of the error between the frequency response of the filter and the desired ideal response and forming a mathematical optimization problem such that it becomes solvable by the linear programming technique. This made the solution of the original problem practicable and efficient. Results in [19] portray that the optimal L_1 filters outcomes a flatter response in the passband and stopband than those of the L_1 and L_∞ filters, while retain a transition band which is comparable to that of the least-squares. Applying the mathematical theory of L_1 filters [15], it was demonstrated that the error function is differentiable, the Hessian matrix was deduced, condition for uniqueness was expressed and a modified globally convergent Newton algorithm was proposed to calculate the optimal filter. Further, it states that the uniqueness usually holds, and even when it does not, fast convergence will be observed.

These classical methods are related with some drawbacks due to which their computational cost increases with slow convergence rate and requires a handful experience for the tuning of filter parameters. FIR filter design being a multi-modal optimization problem, it requires a continuous and differentiable objective function. These techniques cannot optimize a non-uniform, non-differentiable, non-linear, multi-dimensional error fitness function, hence cannot converge to the global minimum results and usually diverge same local sub-optimal solution [20]. They have high sensitivity towards initial points as the number of solution parameters get increased as a result, their capability of searching decreases with an increased problem space. They also demand multiple runs to acquire optimized solutions. This necessitates algorithms with better control of parameters, fast and global convergence. This evolved to the design methods based on heuristic optimization algorithms.

In past research it is found that most of the evolutionary methods for the optimization of digital filters are computed with a differentiable fitness function such as least-square [21−25]. One of the such technique is GA which is developed by Holland [26]. It is a highly flexible population based bio-inspired global optimization technique, inspired by the Darwin's “Survival of the Fittest” and is employed for filter designing in the work reported in [27−30]. Other such algorithms used for finding the optimal filter parameters includes simulated annealing (SA), inspired from annealing in metallurgy [31]; differential evolution (DE) which is a randomized stochastic search technique based on reverse genes [32−34]; bat algorithm is based on the echolocation behavior of bats [35]; particle swarm optimization (PSO) simulates the behavior of bird flocking or fish schooling [24,36]. Filters designed with the above algorithms comprises of more ripples in the passband. To obtain a flatter passband and higher attenuation in the stopband, a novel fitness function based on the L_1 norm is defined. Finally the probabilistic optimization technique RCGA is incorporated with L_1 method to get the global solution with faster convergence.

In this paper the capability to approximate filter in L_1 sense and optimizing using RCGA fitted with L_1-norm is investigated for solving the *N*th order digital FIR filter design problem. The multimodal objective function is chosen in L_1 sense under the constraints of differentiability and uniqueness in solution. The RCGA is employed to obtain nearly best solution in the designing of FIR HP filter. A good and comprehensive simulations results and their statistical analysis are showcased to justify the effectiveness of the algorithm.

The paper is organized as follows: Section 2 formulates the FIR filter design problem using the L_1 fitness function. In Section 3, the RCGA techniques using the L_1 fitness function, employed for designing the FIR filters is presented. Section 4 describes the linear phase FIR HP filter design examples along with the result analysis and comparative outcomes. Finally, the conclusions of the proposed work are highlighted in Section 5.

2. Problem formulation

The digital optimal FIR filter design procedure is based on the L_1-error approximation. The technique involves the evaluation of a weighted error function. The coefficients of the filter are then determined so as to minimize the absolute error that occurs. For the optimal design of *N*th order FIR HP filter, the filter impulse response $h(n)$, $0 \le n \le N$, is approximated to the ideal frequency response, $H_{id}(e^{j\omega})$ specified as

$$H_{id}\left(e^{j\omega}\right) = \begin{cases} 0, & \omega \in [0, \omega_c) \text{ stopband} \\ 1, & \omega \in [\omega_c, \pi] \text{ passband} \end{cases} \quad (1)$$

The frequency response of the approximating filter, $H(e^{j\omega})$ obtained by computing the discrete time Fourier transform (DTFT) of filter impulse response, $h(n)$ is defined as

$$H\left(e^{j\omega}\right) = \sum_{n=0}^{N} h(n) e^{-j\omega n} \quad (2)$$

Considering Type-I linear phase FIR filter with odd length and symmetric coefficient, $\{h(n) = h(N-n), 0 \le n \le N\}$, the amplitude response is given by [2,37].

$$H_r\left(e^{j\omega}\right) = h[M] + 2\sum_{n=1}^{M} h[M-n]\cos(\omega n) \quad (3)$$

where $M = N/2$ and $H_r(e^{j\omega})$ is the real valued function. Since $H_{id}(e^{j\omega})$ is zero-phase, approximating it by $H(e^{j\omega})$ is equivalent to approximating it by $H_r(e^{j\omega})$, and adding a delay of *M*-taps to $H_r(e^{j\omega})$ to make $H(e^{j\omega})$ causal.