European Journal of Operational Research 216 (2012) 113–126

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Production, Manufacturing and Logistics

Solving the two-echelon location routing problem by a GRASP reinforced by a learning process and path relinking

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ARTICLE INFO

Article history:
Received 23 July 2010
Accepted 19 July 2011
Available online 28 July 2011

Keywords:
Two-echelon location-routing problem
Vehicle routing
Facility location
GRASP
Learning process
Path relinking

ABSTRACT

The two-echelon location-routing problem (LRP-2E) arises from recent transportation applications like city logistics. In this problem, still seldom studied, first-level trips serve from a main depot a set of satellite depots, which must be located, while second-level trips visit customers from these satellites. After a literature review on the LRP-2E, we present four constructive heuristics and a hybrid metaheuristic: A greedy randomized adaptive search procedure (GRASP) complemented by a learning process (LP) and path relinking (PR). The GRASP and learning process involve three greedy randomized heuristics to generate trial solutions and two variable neighbourhood descent (VND) procedures to improve them. The optional path relinking adds a memory mechanism by combining intensification strategy and post-optimization. Numerical tests show that the GRASP with LP and PR outperforms the simple heuristics and an adaptation of a matheuristic initially published for a particular case, the capacitated location-routing problem (CLRP). Additional tests on the CLRP indicate that the best GRASP competes with the best metaheuristics published.

1. Introduction and literature review

The *two-echelon location-routing problem* (LRP-2E) plays an important role in transportation networks and particularly in city logistics, for two main reasons. Firstly, the location of intermediate depots (satellites) near large cities allows considerable savings compared to direct deliveries from one main depot. Secondly, more and more municipalities wish to reduce traffic in their city centre, by creating peripheral logistic platforms, from which smaller vehicles are allowed to go downtown. Several cities such as London and Rome have advanced projects in this area.

In Taniguchi and Thompson [33] and Crainic et al. [6], multi-level distribution systems are considered as a tool to reduce urban congestion and increase mobility, while Crainic et al. [7] propose general models for multi-level urban logistics. More generally, we can consider the location-routing problem in the context of N-echelon transportation systems, called LRP-NE. Gonzalez-Feliu [14] presents certain main concepts of multi-echelon distribution with crossdocks and a unified notation for the LRP-NE.

Contrary to the VRP, the literature on location and/or routing problems in multi-echelon networks is still scarce. To our knowledge, the only works on the LRP-2E considered in this paper can be credited to Jacobsen and Madsen [18] and to Madsen [23]. A newspaper delivery system in Denmark consisting of 4500 customers is solved by three constructive heuristics called TTH, ALA-SAV and SAV-DROP. In these early studies, a satellite may be located at any customer, no local search is used, and the heuristics are tested on a single real instance.

The LRP-2E is closely related with other variants of multi-echelon transportation problems. Gonzalez-Feliu [13] recently proposed a vehicle flow model and an exact column generation approach for the *two-echelon vehicle routing problem* (VRP-2E), in which there is no fixed cost for using a satellite depot. It allows finding the optimum for small instances (20 customers) but the distance between lower and upper bounds becomes excessive for 50 customers. Crainic et al. [8] proposed a fast cluster-based heuristic method. Although it can obtain better upper bounds for instances with 50 customers, the gap between the two bounds is still significant. This method was used to realize a satellite location analysis by the same authors [9].

Semet and Taillard [31] solved a real-life vehicle routing problem in Switzerland: the *truck and trailer routing problem* (TTRP). Its main application is the delivery of goods to groceries in areas having limited accessibility, such as city centres or narrow valleys. The TTRP can be viewed as a kind of VRP-2E, with some specific features. The depot has two types of trucks, with or without trailers. Some stores may be supplied directly, by a truck with trailer. The trailer may be detached at some stores, which play the role of satellites, and second level trips are performed by the truck alone to reach customers located on narrow roads. Hence, there are two types of routes. The first type is classical: a route begins at the depot, serves a subset of stores and returns to the depot.

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doi:10.1016/j.ejor.2011.07.030

The second one has two levels like in the VRP-2E: from the main depot, a route visits a subset of stores (satellites), where the trailer is provisionally dropped to perform second-level trips. This real application was solved by a cluster-first route-second heuristic followed by a tabu search. The TTRP was also studied by Scheuerer [30] , who proposed a tabu search.

Recently, another TTRP version called *single truck and trailer routing problem with satellite depots* (STTRPSD) and found in milk collection was studied by Villegas et al. [35]. In the STTRPSD, a truck (tanker) with a detachable tank-trailer must visit a set of farms. A set of possible parking locations along main roads, called trailer-points, can be used to drop the trailer. Each farm must be serviced by exactly one second-level route (performed by the truck alone), starting and ending at one trailer point. This is a particular case of LRP-2E with one route at the first level, uncapacitated satellites and no opening costs. A GRASP and an evolutionary local search (ELS) were developed to solve this problem. The same authors also proposed a GRASP with evolutionary path relinking for the TTRP [36].

The case of customers with large demands also raises multi-echelon facility location problems, but with direct (truckload) routes. For instance, Tragantalerngsak et al. [34] proposed Lagrangian heuristics for the *two-echelon capacitated facility location problem* (CFLP-2E), where plants and warehouses must be selected in the two first layers of a three-layer distribution system. Li et al. [22] developed a scatter search for a multi-product version in which a shipment may transit via several warehouses, to be located. Two-echelon location problems were also studied by Kaufman et al. [20] and Ro and Tcha [29], who independently proposed branch-and-bound algorithms to locate plants and warehouses. More recently, Gendron and Semet [12] considered a *multi-echelon capacitated location–distribution problem* with four layers of nodes: hubs, depots, satellites and customers. They proposed a mathematical model, relaxation methods and a variable neighbourhood heuristic. However, direct deliveries are used between two adjacent layers and there is no routing aspect.

The *vehicle routing problem with satellite facilities* (VRPSF) has been investigated for instance by Jordan and Burns [19], Bard et al. [2], Angelelli and Speranza [1] and Crevier et al. [10]. Each vehicle must start and end its route at the same depot, like in the multi-depot VRP, but it may refill at any depot. Moreover, the way the depots are supplied is not addressed.

Finally, we must mention the *location-routing problem* (LRP), in which the satellites are not connected by first-level trips. Wu et al. [38] studied the LRP with a limited number of vehicles and tested a simulated annealing algorithm, with a tabu list to avoid cycling, on instances with 12–85 customers. Recently, several effective metaheuristics for the LRP with capacitated depots (CLRP) and unlimited fleet were applied to instances with up to 200 customers and 10 depots: a GRASP [26], a memetic algorithm [27] and a matheuristic [28] by Prins et al., a GRASP hybridized with an evolutionary local search by Duhamel et al. [11], and two exact methods by Belenguer et al. [3] and Contardo et al. [5].

The remainder of this paper is organized as follows. Section 2 defines the problem and introduces a mathematical model for the LRP-2E. Section 3 presents four constructive heuristics and a GRASP complemented by a learning process. Section 4 describes a path relinking procedure and its integration in the GRASP developed previously. The computational evaluation and a conclusion follow in Sections 5 and 6.

2. Problem definition and mathematical model

The LRP-2E can be defined on a weighted and complete undirected graph $G = (V, E, C) \cdot V$ is a set of nodes with one main depot (node 0), a subset V_S of m possible satellite locations (indexed from 1 to m) and a subset $V_C = \{m+1, m+2, \ldots, m+n\}$ of n customers. In the edge-set E, each edge $[i,j]$ models a shortest path in the actual network, with a known travelling cost c_{ij}. A capacity W_i and an opening cost O_i are associated with each satellite $i \in V_S$. Each customer $j \in V_C$ has a known demand d_j. A set of first-level or primary vehicles with capacity Q_1 is available at the main depot. A set of second level or secondary vehicles with capacity Q_2 is shared by the satellites. When used in level 1 (respectively level 2), a vehicle incurs a fixed cost F_1 (resp. F_2) and performs one single route. The size of each fleet is a decision variable. It is assumed that the main depot and the total capacity of satellites can always satisfy the total demand of customers.

The following constraints must hold: each vehicle performs at most one trip; each secondary trip must begin and end at the same open satellite; each customer or open satellite must be served by one single vehicle (no split deliveries); each open satellite must receive enough goods from a primary trip to satisfy the customers of its secondary trips; vehicle and satellite capacities must be respected. The objective function, to be minimized, is the total cost of the system, which includes the costs of primary and secondary trips, the fixed costs of vehicles involved, and the opening costs of selected satellites. The LRP-2E is NP-hard because it generalizes three known NP-hard problems: the two-echelon facility location problem (FLP-2E), if only direct deliveries are allowed, the two-echelon vehicle routing problem (VRP-2E) in which all satellites are already open, and the capacitated location-routing problem (CLRP), if the costs of edges linking two satellites or a satellite and the main depot are null.

Fig. 1 depicts two examples of feasible solutions, with customer and satellite demands in brackets. The one on the right corresponds to a more general case in which customers can be supplied by the main depot. Such deliveries must still be made by secondary vehicles, which are the only ones authorized in city centres. This extension is easily handled by placing a fictitious satellite on the main depot, with a null opening cost and a capacity equal to the total demand.

The LRP-2E can be modelled as a two-index integer linear program (without vehicle indices). Let $V_1 = \{0\} \cup V_S$ be the set of nodes that can be visited by the primary vehicles and $V_2 = V_C \cup V_S$ the nodes which can be visited by secondary vehicles. For any subset of nodes $F \subseteq V$, $\delta(F)$ denotes the subset of edges with one node in F and the other in $V \backslash F$, while $\gamma(F)$ is the subset of edges with both nodes in F. For two disjoint subsets of nodes F and F', $E(F : F')$ denotes the subset of edges with one node in F and the other in F'. For one subset of edges A and a variable u defined on edges, $u(A) = \sum_{[i,j] \in A} u_{ij}$ and $u(E(F : F'))$ is simplified into $u(F : F')$.

The model involves four types of variables. The binary variables z_i, $i \in V_S$, are equal to 1 if and only if satellite i is opened. The binary variables f_{ij}, $i \in V_S$ and $j \in V_C$, are equal to 1 if and only if customer j is assigned to (served by) satellite i. The integer variables $x_{ij} \in \{0, 1, 2\}$ represent the number of times edge $[i,j] \in \gamma(V_2)$ is traversed by a secondary vehicle. The value 2 is only used for a direct trip between a satellite and a customer. Similarly, the integer variables $y_{ij} \in \{0, 1, 2\}$ represent the number of times edge $[i,j] \in \gamma(V_1)$ is traversed by a primary vehicle. The value 2 corresponds to a direct trip between the main depot and a satellite.

Two lower bounds on the number of primary and secondary vehicles are also used. $r_2(F) = \left\lceil \sum_{j \in F} d_j / Q_2 \right\rceil$, $F \subseteq V_C$, is the minimum number of secondary vehicles or routes to serve the customers in subset F. For a subset of satellites $F \subseteq V_S$, the minimum number of primary vehicles (or routes) to supply F depends on the customers assigned to these satellites, i.e, $r_1(F) = \left\lceil \sum_{i \in F} \sum_{j \in V_C} d_j f_{ij} / Q_1 \right\rceil$. In fact, the *ceil* function must be removed in the expression of $r_1(F)$, to avoid making the model non-linear.