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Analysis of thin film flow over a vertical oscillating belt with a second grade fluid

Taza Gul [a], Saeed Islam [a], Rehan Ali Shah [b], Ilyas Khan [d,*], Sharidan Shafie [c], Muhammad Altaf Khan [a]

[a] Department of Mathematics, Abdul Wali K.U Mardan, Khyber Pakhtunkhwa (KPK) Pakistan
[b] Department of Mathematics, U.E.T Peshawar, Khyber Pakhtunkhwa (KPK) Pakistan
[c] Department of Mathematical Sciences, Faculty of Science, Universiti Teknologi Malaysia, 81310, UTM Johor Bahru, Johor, Malaysia
[d] Department of Basic Sciences, College of Engineering Majmaah University, P.O. Box 66, Majmaah 11952, Saudi Arabia

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A B S T R A C T

An analysis is performed to study the unsteady thin film flow of a second grade fluid over a vertical oscillating belt. The governing equation for velocity field with appropriate boundary conditions is solved analytically using Adomian decomposition method (ADM). Expressions for velocity field have been obtained. Optimal asymptotic method (OHAM) has also been used for comparison. The effects of Stocks number, frequency parameter and pressure gradient parameters have been sketched graphically and discussed.

1. Introduction

The flow of thin fluid has many applications in our daily life. In engineering we see their usage in condensers, distillation units and heat exchangers. In geophysical events we see thin fluid films in the forms of drilling mud, heat pipes and debris flow. In biological science thin fluid films coating the airways in the lungs and thin tear films covering the eye.

On the other hand, non-Newtonian fluids in view of their numerous applications in engineering and industry have been widely studied. For example, few of these applications in industry are found in wire and fiber coating, paper production, transpiration cooling and gaseous diffusion. Considerable efforts have been made to study non-Newtonian fluids through analytical and numerical treatment. Rehan et al. [1] studied unsteady flow of a second grade fluid between a wire and die with one oscillating boundary and other stationary.

The physical importance of thin film has been investigated and discussed by various scientists [2–6]. Amongst them, the thin film flow of a power law model liquid falling down an inclined plate was discussed by Miladinova et al. [7], where they observed that saturation of non-linear interaction occurred in a finite amplitude permanent wave. Alam et al. [8] investigated the thin-film flow of Johnson-Segalman fluids for lifting and drainage problems and observed the effects of various parameters on the lift and drainage velocity profiles. In the literature several mathematical models of non-Newtonian fluids have been proposed. One of the well-known model amongst the non-Newtonian fluids is a subclass of differential type fluids known as second grade fluids which has its constitutive equations based on strong theoretical foundations. Based on this motivation, for the present research we have chosen the second grade fluid as non-Newtonian fluid.

In order to solve the real world problems, different numerical, exact and approximate techniques have been used in mathematics, fluid mechanics and engineering sciences [9–11]. Some of the common methods are HAM and OHAM [12,13]. Application of optimal homotopy asymptotic method for solving non-linear

* Corresponding author.
E-mail address: ilyaskhanqau@yahoo.com (I. Khan).
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equations arising in heat transfer was investigated by Marinca et al. [14]. In another investigation Marinca et al. [15] have used optimal homotopy asymptotic method for the steady flow of a fourth-grade fluid past a porous plate. Besides that the thin film unsteady flow with variable viscosity has been investigated by Nadeem and Awais [16]. They have analyzed the effect of variable thermo capillarity on the flow and heat transfer. Khajohnsaksumeth et al. [17] studied the effects of slip boundary conditions on the flow of a non-Newtonian fluid through micro channels. They used modified second-grade fluid model where they represented viscosity and the normal stresses in terms of shear rate. The application of their work is focused on blood flow in the cardiovascular system. Taza Gul et al. [18] used ADM and OHAM for the solution of thin film flow of a third grade fluid on a vertical belt with slip boundary conditions. They analyzed the comparison of these two methods.

The main aim of the present work is to study the effects of oscillation into a thin film flow of an unsteady second grade fluid over a vertical belt using ADM and OHAM [19–24].

2. Basic equations

The constitutive equations for an incompressible unsteady flow are

$$\nabla\cdot\mathbf{u} = 0, \tag{1}$$

$$\rho\frac{D\mathbf{u}}{Dt} = \nabla\cdot\mathbf{T} + \rho\mathbf{g}, \tag{2}$$

where ρ is the constant density, $\mathbf{g}$ is used as force per unit mass, $\mathbf{u}$ is the velocity vector, $\mathbf{L} = \nabla\mathbf{u}$, $D/Dt = \partial/\partial t + (\mathbf{u}\cdot\nabla)$ denotes material time derivative and $\mathbf{T}$ is the Cauchy stress tensor which has the following form for a second grade fluid

$$\mathbf{T} = -p\mathbf{I} + \mu\mathbf{A}_1 + \alpha_1\mathbf{A}_2 + \alpha_2\mathbf{A}_1^2. \tag{3}$$

here $-p\mathbf{I}$ denote spherical stress, α_1 and α_2 are the material constants and $\mathbf{A}_1$, $\mathbf{A}_2$ are the kinematical tensors defined as:

$$\mathbf{A}_1 = (\nabla\mathbf{u}) + (\nabla\mathbf{u})^T, \tag{4}$$

$$\mathbf{A}_n = \frac{D\mathbf{A_{n-1}}}{Dt} + \mathbf{A}_{n-1}(\nabla\mathbf{u}) + (\nabla\mathbf{u})^T\mathbf{A_{n-1}}, \quad n>1. \tag{5}$$

3. Formulation of the lift problem

Consider a wide flat belt moving vertically upward at time $t>0$, the belt is oscillated and translated with constant speed U through a large bath of second grade liquid. The belt carries with itself a layer of liquid of constant thickness δ. Coordinate system is chosen for analysis in which the x-axis is taken parallel to the surface of the belt and y-axis is perpendicular to the belt. Uniform magnetic field is applied transversely to the belt. Assuming the flow is unsteady and laminar after a small distance above the liquid surface layer.

The velocity field for the present flow is defined as:

$$\mathbf{u} = (0, u(x,t), 0) \tag{6}$$

The associated boundary conditions are:

$$\mathbf{u}(x,t) = U + U\Omega cos\omega t \quad \text{at } x=0, \quad \frac{\partial\mathbf{u}(x,t)}{\partial x} = 0 \quad \text{at } x=\delta. \tag{7}$$

Here Ω is used as amplitude and ω is used as frequency of the oscillating belt.

Inserting the velocity field from Eq. (6) in continuity Eq. (1) and in momentum Eqs. (2) and (3), the continuity Eq. (1) satisfies identically and Eqs. (2) and (3) are reduced to the following components of stress tensor

$$T_{xx} = -P + (2\alpha_1+\alpha_2)\left(\frac{\partial u}{\partial x}\right)^2, \tag{8}$$

$$T_{xy} = \mu\frac{\partial u}{\partial x} + \alpha_1\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x}\right), \tag{9}$$

$$T_{yy} = -P + \alpha_2\left(\frac{\partial u}{\partial x}\right)^2, \tag{10}$$

$$T_{zz} = -P, \tag{11}$$

$$T_{xz} = T_{yz} = 0. \tag{12}$$

Making use of Eqs. (8)–(12) into Eq. (3), we get

$$\rho\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial y} + \mu\frac{\partial^2 u}{\partial x^2} + \alpha_1\frac{\partial}{\partial t}\left(\frac{\partial^2 u}{\partial x^2}\right) - \rho g. \tag{13}$$

Introducing the following non-dimensional variables

$$\tilde{u} = \frac{u}{U},\ \tilde{x} = \frac{x}{\delta},\ \tilde{t} = \frac{\mu t}{\rho\,\delta^2},\ \tilde{\omega} = \frac{\omega\,\delta^2\rho}{\mu},\ \lambda = \frac{\delta^2\partial p}{\mu\,U\partial y},\quad S_t = \frac{\delta^2\rho g}{\mu\,U},\ \alpha = \frac{\alpha_1}{\rho\,\delta^2}, \tag{14}$$

where ω is the frequency parameter, λ is non-dimensional pressure gradient parameter, α is the non-Newtonian parameter, t is time parameter and S_t is the Stock's number.

Introducing Eq. (14) into Eq. (13) and dropping out the bar notations, we obtain

$$\frac{\partial u}{\partial t} = -\lambda + \frac{\partial^2 u}{\partial x^2} + \alpha\frac{\partial}{\partial t}\left(\frac{\partial^2 u}{\partial x^2}\right) - S_t. \tag{15}$$

The corresponding boundary conditions (7) are reduced to

$$u_n(0,t) = 1 + \Omega\cos\omega t\ ,\ \ and\ \ \frac{du_n(1,t)}{dx} = 0, \quad n=0, \tag{16}$$

$$u_n(0,t) = 0, \quad \frac{du_n(1,t)}{dx} = 0, \quad n\geq 1. \tag{17}$$

4. Analysis of Adomain decomposition method

The Adomian decomposition method (ADM) is used to decompose the unknown function $u(x,t)$ into a sum of an infinite number of components defined by the decomposition series.

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t), \tag{18}$$