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Design of quadrature mirror filter bank using Lagrange multiplier method based on fractional derivative constraints

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A B S T R A C T

Fractional calculus has recently been identified as a very important mathematical tool in the field of signal processing. Digital filters designed by fractional derivatives give more accurate frequency response in the prescribed frequency region. Digital filters are most important part of multi-rate filter bank systems. In this paper, an improved method based on fractional derivative constraints is presented for the design of two-channel quadrature mirror filter (QMF) bank. The design problem is formulated as minimization of L_2 error of filter bank transfer function in passband, stopband interval and at quadrature frequency, and then Lagrange multiplier method with fractional derivative constraints is applied to solve it. The proposed method is then successfully applied for the design of two-channel QMF bank with higher order filter taps. Performance of the QMF bank design is then examined through study of various parameters such as passband error, stopband error, transition band error, peak reconstruction error (PRE), stopband attenuation (*As*). It is found that, the good design can be obtained with the change of number and value of fractional derivative constraint coefficients.

1. Introduction

Multi-rate systems, especially multi-rate filter banks are widely used in several applications of signal processing such as: in speech processing [1], image processing and telemedicine [2] etc. Basically, multi-rate filter bank structure is divided into two parts: first is analysis section and second is synthesis section. In analysis part, signal is divided into different sub-bands and some specific processes are applied on the sub-bands. In synthesis section, processed sub-bands are combined and signal is reconstructed. Among all the multi-rate filter banks, two-channel filter bank was first used for sub-band coding of speech signal. Two-channel filter bank is fundamental part of multi-rate filter bank and also called as quadrature mirror filter (QMF) bank.

Due to immense application of QMF bank in signal processing, various optimization methods have been developed for the design of optimized QMF bank [3–12]. These optimization methods are categorized as: gradient based optimization methods and nature inspired optimization methods. Gradient based optimization methods basically depend upon the gradient of function like field function method [3], weighted least square method (WLS) [4,5], Levenberg–Marquardt algorithm (LM) [6–8], Quasi-Newton method [8] etc. Gradient based method gives stationary solution where gradient is zero. In [3], a novel approach for QMF bank has been proposed using field function method. In [3], modified field function is introduced for obtaining global minima using various extra constraints on prototype filter and filter bank. A new WLS technique for designing FIR filter, based on neural network, has been developed by researchers recently [4]. Another technique based on neural network was proposed for designing QMF bank without any convergence problem [5]. A different approach for the design of two-channel QMF bank using Marquardt optimization method is given in [6]. Design was further modified using Levenberg–Marquardt (LM) method [7]. Recently, authors in [8] have used Levenberg–Marquardt (LM) and Quasi-Newton (QN) optimization method simultaneously for the effective design of two-Channel QMF bank.

Recently, nature inspired methods are widely used for obtaining global solution of linear and nonlinear problems because stochastic and meta-heuristic nature. Nature inspired methods are basically

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motivated by the natural phenomena's such as biological process: reproduction, mutation and interaction and social behavior: flock of birds, schooling of fish, and intelligence of swarm of bee. Nature inspired methods such as genetic algorithms [9], particle swarm optimization (PSO) [10,11] and differential evolution [12] are associated with global solution. In [9], a signed powers-of-two coefficient perfect reconstruction QMF bank using CORDIC genetic algorithm, which is based on coordinate rotation, is designed. In [10], a QMF bank design by PSO with nonlinear unconstraint optimization is given. QMF design was further improved by modified PSO [11]. In [11], modification is done by hybridization of PSO with Scout Bee form of artificial bee colony (ABC) algorithm. In [12], QMF design was further enhanced using adaptive differential evolution algorithm.

All these approaches are suitable for low order QMF filter bank design and for higher order, efficient design of QMF bank is not possible due to high ripples that occur in passband and stopband of analysis and synthesis filters. In [13], linear finite impulse response (FIR) filter has been designed by fractional derivative at prescribed frequency point, and it gives smoother passband and stopband region than conventional least square and conventional derivative methods which show ability of fractional derivative towards more confined analysis of two-channel QMF bank and can be used to design higher order two-channel QMF bank. However, as the authors of this paper have been able to ascertain on the basis of detailed literature review, very less [14] attempt has been made for the design of two-channel QMF bank by using fractional derivative. This paper, therefore, presents the design of higher order QMF bank using fractional derivative constraints.

Fractional calculus is a branch of mathematics which admits definition of fractional derivative and fractional integral. In fractional calculus, definition of integral and derivative operators are almost same as the conventional definition, only integer order of calculus is changed into fractional order. It means order becomes more generalized in fractional calculus. In term of mathematical framework, fractional derivative is a very old concept, but from application point of view, it is a very novel concept. For function $f(x)$, nth order fractional derivative is defined as

$$D_x^n f(x) = \frac{(d^n f(x))}{(dx^n)} \tag{1}$$

Fractional calculus has been identified as very important mathematical tool in many areas of science and engineering such as basic physics [16], numerical methods [17], material science [18], optical engineering [19], earthquake engineering [20], electric network [21], control system [22], fluid mechanics [23], heat mechanics [24], electromagnetic theory [25] and signal processing [26] etc. Fractional calculus has been exploited in many applications of signal processing, such as design of fractional order differentiator for image sharpening, estimation of fractional noise [26] and estimating the fractional order derivative for a given contaminant signal [27]; design of fractional order multi-scale variation modal to preserve textural information of image, to eliminate staircase effect [28] and for image de-noising [29]; design of more flexible RL and RC circuit by fractional order derivative [30]; development of very low sensitive to noise method by using fractional derivative for identification of blur parameters of the motion blurred image [31]. Fractional derivative is also used to update the coefficients according to prescribed adaptive learning algorithm [32]; to design filter for image contrast enhancement [33]; for the design of fixed fractional delay FIR filter [34] and for efficient design of one and two dimensional linear phase FIR filter [13,35].

The most famous definitions of fractional derivative that have been popularized in the world of fractional calculus are: Riemann–Liouville, M. Caputo, and Grunwald–Letnikov definitions. In first two definitions, fractional derivative is being evaluated with the use of integration. Because of integration, these two fractional derivatives are more computationally complex as compared to Grunwald–Letnikov definition. Due to summation form of Grunwald–Letnikov fractional derivative, this formula became the first choice of computer numerical solver. Because of inherent low complexity, Grunwald–Letnikov definition of fractional derivative has been used in this paper. Mathematical definition of fractional derivative given by Grunwald-Letnikov is [13]:

$$D_x^u f(x) = \frac{(d^u f(x))}{(dx^u)} = \lim_{\Delta\to 0} \sum_{k=0}^{\infty} \frac{(-1)^k C_k^u}{\Delta^u} f(x - k\Delta) \tag{2}$$

where, C_k^u is given by

$$C_k^u = \frac{\Gamma(u+1)}{\Gamma(k+1)\Gamma(u-k+1)} = \begin{cases} 1 & k=0 \\ \dfrac{u(u-1)(u-2)\ldots(u-k+1)}{1.2.3\ldots k} & k \geq 1 \end{cases} \tag{3}$$

and $\Gamma(\cdot)$ is the gamma function.

In Section 2, fractional derivative defined by Eq. (2) will be used to design prototype filter for QMF bank. The paper is structured as follows: In this section, brief introduction of two-channel QMF bank and fractional derivative is given. In Section 2, design formulation of QMF is explains. In Section 3, results obtained by the proposed design method are discussed. Finally, conclusions are made in Section 4.

2. Formulation of design problem

Two-channel filter bank is fundamental multirate filter bank. Recently, different design formulation and theory of two-channel filter bank were proposed by many researchers [3–12]. Two-channel filter bank divides input signal into two subbands using analysis section and after applying some specific or required operation signal reconstructed using synthesis section of two-channel filter bank. Basic block diagram of two-channel filter bank is shown in Fig. 1. Ideally, reconstructed output should be exact replica of the input signal with some delay. Since, different types of distortion occur in QMF filter bank, exact reconstruction is not possible.

Basically, three types of distortion occur in QMF bank: aliasing distortion, phase and amplitude distortion or peak reconstruction error (PRE). Aliasing distortion occurs because of non-idealness (realization of ideal filter is not possible) of filter, which can be eliminated using a suitable design of the synthesis filters. Phase distortion occurs because of nonlinear phase response of the filter, which can be eliminated using linear phase FIR filter. But, peak reconstruction error can never be eliminated completely. In case of a two-channel QMF bank shown in Fig. 1, aliasing distortion is eliminated by [6–8].

$$H_1(z) = H_0(-z), \quad G_0(z) = H_1(-z) \quad \text{and} \quad G_1(z) = -H_0(-z) \tag{4}$$

where, $H_0(z)$ and $H_1(z)$ are analysis filters, while $G_0(z)$ and $G_1(z)$ are the synthesis filters.

From Eq. (4), it can be interpreted that all the filters ($H_1(z)$, $G_0(z)$ and $G_1(z)$) of QMF bank are derived by prototype filter $H_0(z)$. Let prototype filter $H_0(z)$ is a linear phase FIR filter has frequency response

$$H_0\left(e^{j\omega}\right) = e^{-j\omega(N-1)/2} H_0(\omega) \tag{5}$$