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Decision Support

# Location of a semi-obnoxious facility with elliptic maximin and network minisum objectives

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ABSTRACT

This paper considers the problem of locating a single semi-obnoxious facility on a general network, so as to minimize the total transportation cost between the new facility and the demand points (minisum), and at the same time to minimize the undesirable effects of the new facility by maximizing its distance from the closest population center (maximin). The two objectives employ different distance metrics to reflect reality. Since vehicles move on the transportation network, the shortest path distance is suitable for the minisum objective. For the maximin objective, however, the elliptic distance metric is used to reflect the impact of wind in the distribution of pollution. An efficient algorithm is developed to find the nondominated set of the bi-objective model and is implemented on a numerical example. A simulation experiment is provided to find the average computational complexity of the algorithm.



## 1. Introduction

Stricter environmental standards and increasing public awareness compels production engineers and urban designers to pay more attention to sustainability and human health issues when placing new materials or facilities into the environment, especially when they have adverse effects on people. Some examples of these semi-obnoxious facilities are power plants, chemical plants, waste dumps or airports as listed in Melachrinoudis and Xanthopulos (2003). From the pollution point of view, these facilities should be located as far as possible from population centers to minimize the risk of hazards but this will increase the transportation cost, which is not acceptable. This contrasting nature led many researchers to study this problem as a push–pull bi-objective model (Eiselt and Laporte, 1995).

In most studies of push–pull facility location models, researchers have used the same distance metric for both push and pull objectives. But as pointed out in Ohsawa and Tamura (2003), the odor and noise do not spread according to road maps. They also do not distribute equally in different directions but come from the pollution source along the wind direction. Ohsawa and Tamura (2003) presented a bi-objective model with elliptic maximin and rectilinear minisum objectives on the plane. Using elliptic distance that can model dispersion of pollution according to prevalent wind speed and direction made maximin function more realistic. However rectilinear distance does not model realistically the minisum objective, since vehicle movement takes place on a general transportation network more often than along two perpendicular directions according to Manhattan metric.

In this paper we model realistically the problem of locating a single semi-obnoxious facility on a transportation network using two objectives. The first objective is to minimize the total transportation cost between the new facility and the demand points, expressed as a sum of weighted network distances (minisum). The second objective is to minimize the undesirable effects introduced by the new facility, expressed as maximizing the minimum elliptic distance between the new facility and population centers (maximin). So the distance metric for the maximin objective is elliptic and for the minisum objective is shortest path.

Employing the elliptic distance in a network solution space presents a challenge. Although network topology and edge lengths information is sufficient for deriving the shortest path distance between any two points of the network, the Euclidean distance and its generalized form, the elliptic distance, require that the network be embedded on the Euclidean plane for these distances to be expressed. Thus, the use of mixed metrics in this paper forces us to work on both the network space and the Euclidean space.

In the remainder of this paper, we first present a literature review of the semi-obnoxious facility location problem in Section 2, and then formulate the problem as a network minisum-elliptic maximin model in Section 3. In Section 4 we investigate the characteristics of both objectives and introduce two lemmas that enable us to fathom inefficient solutions. An algorithm is presented at the end of Section 4. A numerical example is presented in Section 5 to illustrate the use of the algorithm and computational experiments are

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conducted in Section 6 to demonstrate the polynomial average computational complexity of the algorithm.

## 2. Literature review

All works in the bi-objective semi-obnoxious facility location literature limit their study either on the plane or on the network and the majority of them use the same distance metric for both the pull and push objectives.

Among these studies, planar models dominated the literature in the past four decades. In the first published study, Mehrez et al. (1985) consider a weighted objective function, which combines both rectilinear minimax and maximin objectives. Brimberg and Juel (1998a) develop a model with two minisum objectives in Euclidean space and minimize the weighted sum of these two objectives, with weights adding to one. They use the traditional Euclidean distance minisum objective to represent the transportation cost and another minisum objective of the negative powers of Euclidean distances, to represent an aggregate undesirable effect. They propose a heuristic based on a branch and bound algorithm called Big Square Small Square (BSSS) to solve their model. Melachrinoudis (1999) uses Fourier–Motzkin elimination to solve the bi-objective rectilinear minisum–maximin model, taking advantage of the small size of the resulting linear bi-objective problems. Plastria and Carrizosa (1999) use an approach based on locating an open disk in a network or plane. The first objective is to reduce the affected population, in other words to minimize the coverage, and the second objective is to increase the radius of the disk of affected points, in order to raise the level of legal protection or to use less costly technology for the facility. Ohsawa (2000) proposes an algorithm based on nearest point and farthest point Voronoi diagrams to find the efficient set of a Euclidean maximin–minimax location problem. Melachrinoudis and Xanthopulos (2003) solve a Euclidean minisum–maximin model using the Voronoi diagram and the K-K-T optimality conditions. Yapicioglu et al. (2007) use Particle Swarm Optimization (PSO) metaheuristic to solve a bi-objective model. Their first objective is the traditional weighted minisum, while the second objective is minimizing the total undesirable effect of the new facility to inhabitants, measured as a piecewise linear function of their distances. Karasakal and Nadirler (2008) propose a three-phase algorithm, called Interactive Generalized Big Square Small Square (IGBSSS) method, for the solution of a rectilinear maximin–minisum facility location problem. After fathoming the inefficient parts of the feasible region in the first two phases, named Rough cut and Precise cut, in the third phase they suggest an interactive search in the remaining regions with the involvement of a DM in order to find the most preferred efficient solution.

Unlike planar models, there are not many network models for the bi-objective semi-obnoxious facility location problem. Hamacher et al. (2002) formulate a negatively correlated maxisum–minisum semi-obnoxious facility model with weighted shortest path network distances and generalize their result to incorporate maximin and minimax objectives. Skriver and Andersen (2003) consider the model of Brimberg and Juel (1998a) in two separate cases of planar and network solution space. They employ an adaptation of BSSS, namely, Edge Dividing (ED) method, to solve the network case. They divide each edge in two sub-edges and calculate the bounds on the objective function values on each sub-edge. If the bounds of a sub-edge are dominated by a sample set of objective values, then the sub-edge is dominated. For a comprehensive review of multi-objective facility location problems and semi-obnoxious facility location problems see Farahani et al. (2010) and Melachrinoudis (2011), respectively.

We only found four papers in the bi-objective semi-obnoxious facility location literature where different distance metrics are used for different objectives. In their bi-objective model, Brimberg and Juel (1998b) use as first objective the minisum of weighted distances with an arbitrary norm and as second objective the traditional weighted Euclidean maximin. They present two reformulations with two different solution methods, but only one of them is guaranteed to find the complete set of efficient solutions. In the second paper, Blanquero and Carrizosa (2002) consider the traditional Euclidean maximin for the obnoxious objective but for transportation cost, instead of using the traditional minisum, they define a general cost function to be minimized. They offer an algorithm based on decomposing the problem to a set of subproblems, defined over Voronoi cells, to find the $\varepsilon$-efficient set of the problem. In the third paper, Skriver and Andersen (2003) consider the model presented in Brimberg and Juel (1998a) but instead of Euclidean distances, they use different norms for distance metrics of the objectives. For finding the efficient set, instead of combining the two objective functions, they provide an approximation algorithm for the bi-objective model based upon BSSS. In the fourth paper, Ohsawa and Tamura (2003) formulate the case of elliptic maximin and rectilinear minisum problem and solve it by decreasing the dimension of decision space from two to one and applying a four-step algorithm to this new space. The solution space in all models above is the plane. Although the use of network distances for transportation cost, and planar distances for the obnoxious objective function has been suggested as future research in various papers in the past two decades (Boffey and Karkazis, 1995; Skriver and Andersen, 2003; Yapicioglu et al., 2007), there is no published work in the location literature where planar and network distances are practically combined in a bi-objective model.

## 3. Problem formulation

Let $G(V, E)$ be a finite, connected, undirected and weighted graph representing a transportation network with node set $V$ and edge set $E$. The transportation network is within a bounded geographical area, such as a county or state. The node set $V$ is the union of three sets, (i) the set of boundary points $H$ that represent intersections of roads with the boundary of the geographical area, (ii) the set of demand points $P$ representing existing facilities, and (iii) the set of nodes that represent road intersections $R$, i.e., $V = H \cup P \cup R$. For example, $P = \{1, 2, 3, 4\}$, $R = \{5, 6\}$, and $H = \{7, 8, 9, 10\}$ in Fig. 1a. Each node $i \in P$ has a non-negative weight $w_i$, representing its demand. Elements of edge set $E$ are unordered pairs of distinct nodes that represent roads. Two nodes $p$ and $q$ in $V$ are adjacent if $(p, q) \in E$. Each edge $(p, q)$ has a positive weight $c(p, q)$ representing road length.

Consider a Euclidean plane that contains the geographical area on which population centers are defined as points $C_l = (a_l, b_l)$, $l \in L$. Let us now embed the graph $G(V, E)$ onto this plane so that the elliptic distance can be defined between a point of the graph and a population center which is not necessarily located on the graph. This is necessary in order to quantify the impact of the semi-obnoxious facility as a function of distance. Each node $i \in V$ can be also considered a point with coordinates $x_i$ and $y_i$ on the Cartesian plane, or $U_i = (x_i, y_i)$. In other words, while $i$ represents a node on graph $G$, $U_i$ represents its geographical location on the plane. Some demand points may be at the same geographical location as population centers. Similarly, corresponding to each edge $(p, q) \in E$, there is a road which connects two points $U_p$ and $U_q$ on the plane. As some roads such as (2, 5) and (1, 5) in Fig. 1a, are arbitrary curves rather than straight lines, calculating the elliptic distance between a point on those roads and a population center is not straightforward. To make our calculations easy, we linearly approximate the roads of the network. This will introduce some artificial nodes (e.g. nodes 11 and 12 in Fig. 1b) and new edges