Engineering Science and Technology, an International Journal 17 (2014) 247–259

HOSTED BY

ELSEVIER

Contents lists available at ScienceDirect

Engineering Science and Technology, an International Journal

journal homepage: http://www.elsevier.com/locate/jestch

Full length article

Laminated composite stiffened shallow spherical panels with cutouts under free vibration – A finite element approach

Sarmila Sahoo

Department of Civil Engineering, Heritage Institute of Technology, Kolkata 700107, India

ARTICLE INFO

Article history:
Received 23 May 2014
Received in revised form
30 June 2014
Accepted 14 July 2014
Available online 16 September 2014

Keywords:
Laminated composites
Stiffened shallow spherical shell panel
Cutout
Free vibration
Finite element

ABSTRACT

In this paper, finite element method has been applied to analyze free vibration problems of laminated composite stiffened shallow spherical shell panels with cutouts employing the eight-noded curved quadratic isoparametric element for shell with a three noded beam element for stiffener formulation. Specific numerical problems of earlier investigators are solved and compared to validate the present formulation. Moreover, free vibration problem of stiffened shallow spherical shell panels with different size and position of the cutouts with respect to the shell centre for different edge constraints are examined to arrive at some conclusions useful to the designers. The results are presented in the form of figures and tables. The results are further analyzed to suggest guidelines to select optimum size and position of the cutout with respect to shell centre considering different practical constraints.

1. Introduction

Finite element method has become an efficient tool to analyze complex structures and materials. The dynamic analysis of shell structures, which may have complex geometry and arbitrary loading and boundary conditions, can be solved efficiently by the finite element method, even including cutouts in shells. Laminated composites are increasingly being used nowadays in aerospace, civil, marine and other related weight-sensitive engineering applications requiring high strength-to-weight and stiffness-to weight ratios. Some of the structural components of aircrafts, missile and ship structures can be idealized as composite shell panels. The most commonly encountered types of shell structures are shells of revolution. Amongst all the existing shells of different geometry, spherical shells, shells of revolution with curved meridian, have many applications in all types of industries. Spherical pressure vessels are a common phenomenon in the chemical and process industries. Quite often, to save weight and also to provide a facility for inspection, cutouts are provided in these panels. In practice the margin of the cutouts must be stiffened to take account of stress concentration effects. Also, there can be some instruments directly fixed on these panels, and the safety of these instruments can be dependent on the vibration characteristics of the panels. Hence free vibration studies on laminated composite spherical shell panels with cutouts are of interest to both structural engineers and materials scientists.

Different computational models for laminated composites were proposed by Kapania [1], Noor and Burton [2], and Reddy [3]. Chao and Reddy [4] reported on the dynamic response of simply supported cylindrical and spherical shells. The transient response of spherical and cylindrical shells with various boundary conditions and loading was reported by Reddy and Chandrashekhara [5]. Chao and Tung [6] presented an investigation on the dynamic response of axisymmetric polar orthotropic hemispherical shells. Later free vibration study of doubly curved shells was done by Qatu [7], Liew and Lim [8-9], Chakravorty et al. [10–12], Shin [13] and Tan [14]. Kant et al. [15] solved problems of a clamped spherical and simply supported cylindrical cap under external pressure. Sathyamoorthy [16] reported the nonlinear vibration of moderately thick orthotropic spherical shells. Later in 1997, Gautham and Ganesan [17] reported free vibration characteristics of isotropic and laminated orthotropic spherical caps while Chia and Chia [18] reported nonlinear vibration of moderately thick antisymmetric angle ply shallow spherical shell. Free vibration of curved panels with cutouts was reported by Sivasubramonian et al. [19]. Qatu et al. [20] reviewed the work done on the vibration aspects of composite shells between 2000 and 2009 and observed that most of the researchers dealt with closed cylindrical shells.

E-mail addresses: sarmila.sahoo@gmail.com, sarmila_ju@yahoo.com.
Peer review under responsibility of Karabuk University.

http://dx.doi.org/10.1016/j.jestch.2014.07.002

Notations

a,b	length and width of shell in plan
a',b'	length and width of cutout in plan
b_{st}	width of stiffener in general
b_{sx}, b_{sy}	width of x- and y-stiffeners respectively
d_{st}	depth of stiffener in general
d_{sx}, d_{sy}	depth of x- and y-stiffeners respectively
$\{d_e\}$	element displacement
E_{11}, E_{22}	elastic moduli
G_{12}, G_{13}, G_{23}	shear moduli of a lamina with respect to 1, 2 and 3 axes of fibre
h	shell thickness
M_x, M_y	moment resultants
M_{xy}	torsion resultant
np	number of plies in a laminate
N_1–N_8	shape functions
N_x, N_y	inplane force resultants
N_{xy}	inplane shear resultant
Q_x, Q_y	transverse shear resultant
R_{xx}, R_{yy}, R_{xy}	radii of curvature of shell
u, v, w	translational degrees of freedom
x, y, z	local co-ordinate axes
X, Y, Z	global co-ordinate axes
z_k	distance of bottom of the kth ply from mid-surface of a laminate
α,β	rotational degrees of freedom
ε_x, ε_y	inplane strain component
γ_{xy}, γ_{xz}, γ_{yz}	shearing strain components
ν_{12}, ν_{21}	Poisson's ratios
ξ, η, τ	isoparametric co-ordinates
ρ	density of material
σ_x, σ_y	inplane stress components
τ_{xy}, τ_{xz}, τ_{yz}	shearing stress components
ω	natural frequency
$\overline{\omega}$	non-dimensional natural frequency $=\omega a^2(\rho/E_{22}h^2)^{1/2}$

Other shell geometries have also been investigated. Among those conical shells and shallow shells on rectangular, triangular, trapezoidal, circular, elliptical, rhombic or other planforms are receiving considerable attention. Shallow spherical shells also received some attention. Wang et al. [21] studied wave propagation of stresses in orthotropic thick-walled spherical shells. Lellep and Hein [22] did an optimization study on shallow spherical shells under impact loading. Dai and Wang [23] analyzed stress wave propagation in laminated piezoelectric spherical shells under thermal shock and electric excitation. Dynamic stability of spherical shells was studied by Ganapathi [24] and Park and Lee [25]. Recently, Kumar et al. [26-29] considered finite element formulation for shell analysis based on higher order zigzag theory. Vibration analysis of spherical shells and panels both shallow and deep has also been reported for different boundary conditions [30–33]. A complete and general view on mathematical modeling of laminated composite shells using higher order formulations has been provided in recent literature [34–36]. However shallow spherical shell panels on rectangular or circular planform (spherical cap) with cutout (stiffened along the margin) are missing in the existing literature. Accordingly, the present endeavor focuses on the free vibration behavior of composite shallow spherical shell with cutout (stiffened along the margin) with concentric and eccentric cutouts, and considers the shells to have various boundary conditions.

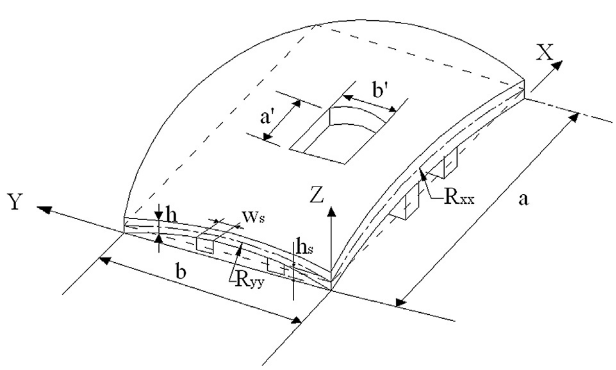

Fig. 1. Spherical shell panel with a concentric cutout stiffened along the margins.

2. Mathematical formulation

A laminated composite spherical shell of uniform thickness h (Fig. 1) and radius of curvature R_{xx} and R_{yy} is considered. Keeping the total thickness the same, the thickness may consist of any number of thin laminae each of which may be arbitrarily oriented at an angle θ with reference to the x-axis of the co-ordinate system. The constitutive equations for the shell are given by (Definitions of symbols used are given in list of notations):

$$\{F\} = [E]\{\varepsilon\} \tag{1}$$

where,

$$\{F\} = \{N_x, \quad N_y, \quad N_{xy}, \quad M_x, \quad M_y, \quad M_{xy}, \quad Q_x, \quad Q_y\}^T,$$

$$\{\varepsilon\} = \left\{\varepsilon_x^0, \quad \varepsilon_y^0, \quad \gamma_{xy}^0, \quad k_x, \quad k_y, \quad k_{xy}, \quad \gamma_{xz}^0, \quad \gamma_{yz}^0\right\}^T,$$

and

$$[E] = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} & 0 & 0 \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} & 0 & 0 \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} & 0 & 0 \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} & 0 & 0 \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} & 0 & 0 \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & S_{11} & S_{12} \\ 0 & 0 & 0 & 0 & 0 & 0 & S_{12} & S_{22} \end{bmatrix}$$

The detailed expressions of the elements of the elasticity matrix are available in several references including Vasiliev et al. [37] and Qatu [38]. The elements of the stiffness matrix $[E]$ are defined as.

$$A_{ij} = \sum_{k=1}^{np} (Q_{ij})_k (z_k - z_{k-1}), \; B_{ij} = \frac{1}{2}\sum_{k=1}^{np} (Q_{ij})_k (z_k^2 - z_{k-1}^2),$$

$$D_{ij} = \frac{1}{3}\sum_{k=1}^{np} (Q_{ij})_k (z_k^3 - z_{k-1}^3) \; i,j = 1,2,6,$$

and

$$S_{ij} = \sum_{k=1}^{np} F_i F_j (G_{ij})_k (z_k - z_{k-1}) \; i,j = 4,5.$$