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Innovative Applications of O.R.

Investment in electricity networks with transmission switching ☆

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We consider the application of Dantzig-Wolfe decomposition to stochastic integer programming problems arising in the capacity planning of electricity transmission networks that have some switchable transmission elements. The decomposition enables a column-generation algorithm to be applied, which allows the solution of large problem instances. The methodology is illustrated by its application to a problem of determining the optimal investment in switching equipment and transmission capacity for an existing network. Computational tests on IEEE test networks with 73 nodes and 118 nodes confirm the efficiency of the approach.

1. Introduction

In this paper we consider economic dispatch models for wholesale electricity supply through an AC transmission network as discussed in e.g. [4]. These models typically make use of a DC-load flow assumption in which reactive power is ignored, line resistance is assumed to be small in comparison to reactance, and voltage magnitudes are treated as constant throughout the system. In such models, Kirchhoff's laws are used to determine the flow on each line. The *voltage* law states that power flow on a transmission line is proportional to the difference in voltage phase angles at each endpoint, and the *current* law states that the total power flowing from the network into any location matches the demand minus supply at this point. Thus, given the optimal dispatch and demand for a tree network, the power flow is uniquely determined by the current law. The voltage phase angles that generate this flow can be uniquely determined up to an additive constant by applying the voltage law.

Most electricity transmission networks are designed as meshed networks (with cycles) for security reasons, so that if any line fails, the power can still flow from source to destination by alternative paths. When the network contains cycles, the voltage law and current law must be applied simultaneously to determine the line flows and voltage angles from the dispatch of flow and generation. The presence of cycles places additional constraints on the line flows that are absent in tree networks. In particular, for each cycle in a network the sum of voltage angle differences (with respect to the direction) around the cycle must equal zero. Hence, each cycle in the network gives rise to one additional constraint on the line flows. This leads to a paradox (see e.g. [2]) in which adding a new line to a transmission network might increase the cost of supplying electricity, even if the cost of the line itself is zero.

Based on these observations, it is easy to see that it may be beneficial in mesh networks to take some lines out of operation — to either decrease system cost or increase reliability [9,17]. The process of taking out lines and bringing them back in is done by opening (respectively closing) a switch at the end of the line and is referred to as *switching*.

The emphasis of this paper is on the use of switching of electricity transmission networks for economic benefit, although security constrained dispatch with post contingency corrective rescheduling (including switching) [15] is considered as a special case. We acknowledge that switching for economic benefit is not common practice in the industry today. However, with increased focus on intermittent and distributed energy sources as well as efficient operation of energy systems this area seems to have a large potential in the future.

Recent interest in renewable intermittent energy sources and the call for intelligent transmission networks or smart grids have spurred a renewed interest in switching problems. Fisher et al. presents in [7] the problem of optimal switching of transmission elements in an electricity transmission network to minimize the delivered cost of energy. They propose a mixed-integer program to solve the DC-load flow economic dispatch model with switching decisions in a single time period. They note that the problem is

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NP-hard. Results are provided for a 118-node network with 186 transmission lines. Hedman et al. [12] extends the model to consider reliability of the network. Reliability constraints are added to the problem to ensure that any line failure will not lead to an infeasible dispatch of generation. They note that in some cases adding reliability constraints increases the value of switching.

In [11] Hedman et al. discuss a decomposition algorithm to solve the transmission switching problem with unit commitment decisions made heuristically over 24 time periods. It is noted that adding transmission switching may yield a cheaper unit commitment plan than what could be achieved without switching. In this model, it is assumed that a technology is available that makes it possible to switch lines instantaneously. That is, a line may be switched automatically from one moment to the next without delay. In this case, switching out lines will (in theory) not affect system security (disregarding failures on switching equipment), since all lines may be switched back in immediately, in case of any failure in the system. Khodaei and Shahidehpour [13] describe a Benders decomposition of the security constrained unit commitment problem with transmission switching that outperforms an integrated MIP-model, and Khodaei et al. [14] provide a Benders decomposition approach for solving capacity expansion problems in electricity networks with active transmission switching.

The solution of the large-scale mixed-integer programming problems that arise when switching is considered remains a challenging obstacle to their implementation in practice. Most of the literature in this area has focused on demonstrating the savings in cost that can be made by transmission switching, while acknowledging that there are still computational hurdles to be overcome when solving large real-life instances. Fisher et al. [7] were unable to prove optimality of transmission switching in the IEEE 118-bus network with a single scenario and unrestricted number of open lines. The heuristic approach presented by Hedman et al. [11] for the transmission switching and unit commitment problem with security constraints is unable to prove optimality for the IEEE 73-bus network over 24 time periods — even with extensive computer resources. Khodaei and Shahidehpour [13] limits the space of switchable lines to find solutions to the security constrained unit commitment problem with transmission switching using Benders decomposition. Even when the single-scenario problems are restricted to allowing a small number of switches, these are sufficiently hard to make a multi-period or multi-scenario model intractable.

Making it possible to switch lines instantaneously often requires that some hardware is installed in the network. Firstly, a switch needs to be installed at the line. Secondly, communications equipment between the switch and operating control center is required to ensure automatic remote operation of the switch. Moreover, the ability to profitably switch lines out might be enhanced by adding new transmission lines to the network to absorb increases in flow.This leads to a two-stage stochastic integer programming problem of determining an optimal capital provisioning plan that will satisfy demand almost surely at least expected cost. Note, that even though the fixed cost of enabling a line to be switched instantaneously may be small (e.g. if the switch is already present and only communication equipment needs to be installed) it may not be worthwhile to enable switching on all lines (unless this cost is 0 for all lines), since some lines may never be switched.

In this paper we show how one can attack the stochastic capital provisioning problem using Dantzig-Wolfe decomposition [6] and column generation to give provably optimal or close to optimal solutions. Our approach is based on the approach of Singh et al. [19] for determining optimal discrete investments in the capacity of production facilities. They proposed a split-variable formulation and Dantzig-Wolfe reformulation resulting in a sub-problem for each node in the scenario tree, and showed how this could enable the solution of previously intractable instances of capacity planning problems for electricity distribution networks. Our contribution in this paper is to show how this methodology applies to a transmission switching model, to enable their solution in settings where there are many scenarios representing future uncertainty. With a limitation on the number of switches used in each scenario, the decomposition approach enables us to solve IEEE test problems with up to 256 scenarios, which appears to be well beyond the capability of competing methods.

We begin the paper by recalling a mixed-integer programming formulation for transmission switching based on the model in [7]. In Section 3, we address the problem of the planning of transmission networks under uncertainty considering both installation of switches and line capacity expansions. In particular, we consider a two-stage stochastic program in which the first-stage decisions concern the investments in switch equipment and line capacity, while the second stage models operational decisions in different scenarios. The model is reformulated using Dantzig-Wolfe decomposition, and solved using column generation. In Section 4 we study the structure of the master problem in order to provide some insights into the strength of the decomposition. We show that the master problem has naturally integer optimal solutions in some circumstances, and provide counter examples where this is not true. Computational results of the method applied to two standard test problems (the IEEE 73-bus network and the IEEE 118-bus network) are presented in Section 5. We then draw some general conclusions about the effectiveness of the approach.

2. Optimal transmission switching

We model the electricity transmission system as a network where N denotes the set of nodes (or busses) and A denotes a set of arcs representing transmission lines (and transformers) connecting the nodes. Let $\mathcal{T}(i)$ denote the set of arcs incident with node i where i is the head of the incident arc, and let $\mathcal{F}(i)$ denote the set of arcs incident with node i, where i is the tail of the incident arcs. So an arc in $\mathcal{F}(i) \cap \mathcal{T}(j)$ is directed *from* node i *to* node j. Since power flow can flow in both directions in a transmission line we allow these flows to take negative values, indicating power flow in the opposite direction from the arc direction.

Many transmission systems consist of alternating current circuits, interlinked by high voltage direct current links. We shall ignore these interconnections in this paper, and assume that all lines carry alternating current. The methodologies can easily be adapted to treat direct current lines as special cases. Note, that even though we assume all lines to be alternating current lines, the models presented are based on the linear direct current optimal power flow approximation as discussed in the introduction.

Let G be the set of all generating units, where $G(i)$ is the set of generating units located in (and supplying electricity to) node i. For simplicity, we assume that each unit $g \in G$ offers its entire electricity capacity u_g to the system at its marginal cost c_g. (A model in which each unit offers a step supply curve is a straightforward extension.) We denote by q_g the dispatch of power of unit g.

At each node i the demand d_i must be met. Load shedding at node i may be modeled by introducing a dummy generator at each node offering d_i at a penalty price.

Each transmission line $a \in A$ is characterized by its reactance X_a and thermal capacity K_a. The flow on line a is denoted P_a, which can be negative in order to model power flows in the direction opposite to the orientation of a.

A subset of lines $S \subseteq A$ are considered to be switchable. Lines that are switchable may be taken out of operation in any given period of time. For each line $a \in S$, $z_a = 1$ denotes that the line has been switched out (opened), while $z_a = 0$ denotes that the switch is closed.