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Decomposition of technical and scale efficiencies in two-stage production systems

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ABSTRACT

Conventional data envelopment analysis (DEA) models are used to measure the technical and scale efficiencies of a system when it is considered as a whole unit. This paper extends the efficiency measurement to two-stage systems where each stage has one process and all the outputs from the first process become the inputs of the second. An input-oriented DEA model for the first process is developed to separate the process efficiency into the input technical and scale efficiencies, and an output-oriented model is developed for the second process to separate the process efficiency into the output technical and scale efficiencies. Combining the two models, the system efficiency is expressed as the product of the overall technical and scale efficiencies, where the overall technical and scale efficiencies are the products of the corresponding efficiencies of the two processes, respectively. The detailed decomposition allows the sources of inefficiency to be identified.

1. Introduction

Data envelopment analysis (DEA) is a technique for measuring the relative efficiency of a set of decision making units (DMUs) which use multiple inputs to produce multiple outputs. Since the pioneering work of Charnes et al. (1978), hundreds of papers reporting its applications and advancements have been published (see, for example, the review of Cook and Seiford (2009)).

Conventional DEA treats the system as a whole unit in measuring the efficiency; it does not take the operation of the component processes into consideration. Hence, the decision maker has only information on the relative performance of the system, but is unaware of processes which cause system inefficiency. More seriously, the effects of a factor on the system thus obtained can be misleading. This frequently happens when a factor is indirectly related to a phenomenon. The work of Charnes et al. (1986) on army recruitment using a two-stage approach was the first study to discuss this issue. Later applications of the two-stage DEA include those of Schinnar et al. (1990) on mental health care, Lovell et al. (1994) on education, Seiford and Zhu (1999) on banking, Noulas et al. (2001) on non-life insurance, Sexton and Lewis (2003) on baseball, and Wang et al. (1997) on IT. Several models for measuring the efficiency of two-stage systems have also been proposed. Kao and Hwang (2010) classified them into three approaches: independent, connected, and relational. The relational model of Kao and Hwang (2008) has attracted substantial attention.

In the independent approach, each process is treated as an independent system; the system and two process efficiencies are calculated independently using conventional DEA models. In the connected approach (Färe and Grosskopf, 1996, 2000), the operations of the two processes are represented by conventional envelopment constraints in calculating the system efficiency. However, the intermediate products connecting the two processes are handled independently. In contrast, the relational approach requires the intermediate products to use the same set of multipliers in the two processes. This approach is attractive because the system efficiency is the product of the process efficiencies, which is in accord with human intuition. Its theoretical foundation was further consolidated when Chen et al. (2010) derived the production frontier for this approach.

The relational model was developed under the assumption of constant returns to scale (CRS). The idea of Banker et al. (1984) for measuring the technical efficiency under the assumption of variable returns to scale (VRS) cannot be embedded into the relational model. In other words, the property of the overall efficiency (under CRS) being the product of technical efficiency (under VRS) and scale efficiency cannot be obtained in the relational model. One is thus unable to judge whether the inefficiency is caused by unsatisfactory technology or inappropriate scale.

A set of models for calculating the technical and scale efficiencies for two-stage systems is developed in this paper. In addition to the relationship derived by Kao and Hwang (2008), which states that system efficiency is the product of the two process efficiencies,

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the efficiency of each process is further decomposed into the product of technical and scale efficiencies. With more detailed information, the decision maker can make better decisions regarding the performance improvement of a DMU.

The rest of this paper is organized as follows. In the next section, the relational model proposed by Kao and Hwang (2008) is reviewed and the models for calculating the technical efficiency of the two processes are also developed. Section 3 then rearranges the technical and scale efficiencies of the two processes to become those of the system. Finally, in Section 4, some conclusions are drawn based on the discussions and findings.

2. Two-stage system

Suppose that a system is composed of two processes connected in series, as shown in Fig. 1, where DMU j applies inputs X_{ij}, $i = 1,\ldots,m$, to produce intermediate products Z_{pj}, $p = 1,\ldots,q$, in Process 1, which in turn are used by Process 2 to produce outputs Y_{rj}, $r = 1,\ldots,s$. The conventional DEA models ignore the intermediate products in measuring efficiency. One of the most representative models is the BCC model (Banker et al., 1984) which, in input-oriented form, measures the (input) technical efficiency for DMU k via:

$$
\begin{aligned}
T_k^I = \max\ & \left(\sum_{r=1}^{s} u_r Y_{rk} - u_0\right) \Big/ \sum_{i=1}^{m} v_i X_{ik}, \\
\text{s.t.}\ & \left(\sum_{r=1}^{s} u_r Y_{rj} - u_0\right) \Big/ \sum_{i=1}^{m} v_i X_{ij} \leqslant 1, \quad j = 1,\ldots,n, \\
& u_r,\ v_i \geqslant \varepsilon, \quad r = 1,\ldots,s, \quad i = 1,\ldots,m, \\
& u_0 \text{ unrestricted in sign},
\end{aligned} \tag{1}
$$

where u_r and v_i are virtual multipliers and ε is a small non-Archimedean number imposed to avoid ignoring any factor (Charnes et al., 1979). When the term u_0 is omitted, Model (1) becomes the CCR model (Charnes et al., 1978), and the associated efficiency, E_k, is usually called the overall efficiency. The ratio of E_k to T_k^I is the (input) scale efficiency, S_k^I (Banker et al., 1984).

The technical efficiency can also be measured from the output point of view via the following output-oriented BCC model (Banker et al., 1984):

$$
\begin{aligned}
T_k^O = \max\ & \sum_{r=1}^{s} u_r Y_{rk} \Big/ \left(\sum_{i=1}^{m} v_i X_{ik} + v_0\right), \\
\text{s.t.}\ & \sum_{r=1}^{s} u_r Y_{rj} \Big/ \left(\sum_{i=1}^{m} v_i X_{ij} + v_0\right) \leqslant 1, \quad j = 1,\ldots,n, \\
& u_r,\ v_i \geqslant \varepsilon, \quad r = 1,\ldots,s, \quad i = 1,\ldots,m, \\
& v_0 \text{ unrestricted in sign}.
\end{aligned} \tag{2}
$$

The ratio of the CCR efficiency E_k to the output BCC technical efficiency T_k^O is the output scale efficiency S_k^O.

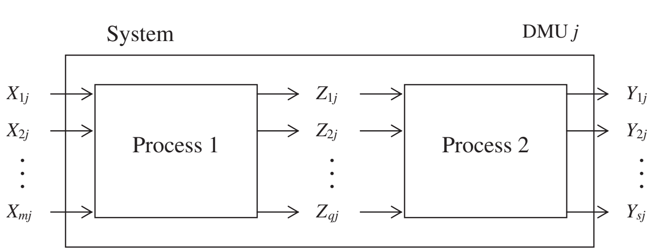

Fig. 1. Two-stage system with inputs X, outputs Y, and intermediate products Z.

2.1. Relational model

In calculating the system efficiency for the two-stage system, Kao and Hwang (2008) proposed a relational model which embeds the operations of the two processes into the CCR model and requires the multipliers w_p associated with the intermediate products Z_p to be the same, regardless of whether Z_p are the outputs of Process 1 or the inputs of Process 2. Their model is:

$$
\begin{aligned}
E_k^S = \max\ & \sum_{r=1}^{s} u_r Y_{rk} \Big/ \sum_{i=1}^{m} v_i X_{ik}, \\
\text{s.t.}\ & \sum_{r=1}^{s} u_r Y_{rj} \Big/ \sum_{i=1}^{m} v_i X_{ij} \leqslant 1, \quad j = 1,\ldots,n, \\
& \sum_{p=1}^{q} w_p Z_{pj} \Big/ \sum_{i=1}^{m} v_i X_{ij} \leqslant 1, \quad j = 1,\ldots,n, \\
& \sum_{r=1}^{s} u_r Y_{rj} \Big/ \sum_{p=1}^{q} w_p Z_{pj} \leqslant 1, \quad j = 1,\ldots,n, \\
& u_r,\ v_i,\quad w_p \geqslant \varepsilon, \quad r = 1,\ldots,s, \quad i = 1,\ldots,m, \\
& \quad p = 1,\ldots,q,
\end{aligned} \tag{3}
$$

where the three sets of constraints correspond to the system, Process 1, and Process 2, respectively. At optimality, the system and process efficiencies are calculated as:

$$
\begin{aligned}
E_k^S &= \sum_{r=1}^{s} u_r Y_{rk} \Big/ \sum_{i=1}^{m} v_i X_{ik}, \\
E_k^1 &= \sum_{p=1}^{q} w_p Z_{pk} \Big/ \sum_{i=1}^{m} v_i X_{ik}, \\
E_k^2 &= \sum_{r=1}^{s} u_r Y_{rk} \Big/ \sum_{p=1}^{q} w_p Z_{pk}.
\end{aligned} \tag{4}
$$

Hence, one has $E_k^S = E_k^1 \times E_k^2$; that is, the system efficiency is the product of the two process efficiencies. Since the first set of constraints in Model (3), $\sum_{r=1}^{s} u_r Y_{rj} - \sum_{i=1}^{m} v_i X_{ij} \leqslant 0$, is the sum of the last two, $\sum_{p=1}^{q} w_p Z_{pj} - \sum_{i=1}^{m} v_i X_{ij} \leqslant 0$ and $\sum_{r=1}^{s} u_r Y_{rj} - \sum_{p=1}^{q} w_p Z_{pj} \leqslant 0$, it is redundant and can be omitted in calculating the efficiencies.

Consider four DMUs A, B, C, and D with the input, intermediate product, and output data shown in columns 2–4 of Table 1, respectively. Fig. 2 shows the two processes in a counterclockwise orientation. On the right side, Process 1 applies input X to produce intermediate product Z, and on the left side, Process 2 applies intermediate product Z to produce final product Y. The straight lines OB^1 and OB^2 passing through the origin on the right and left sides are the production frontiers for Processes 1 and 2, respectively. Note that here two production frontiers are constructed for the two processes, which is different from the approach in Chen et al. (2010) where one production frontier is constructed for the two processes. Since DMU B is the only DMU on both frontiers, it is efficient in both processes, and is consequently efficient for the system. By applying Model (3) and Expression (4), the system and process efficiencies of the four DMUs can be calculated, as shown in Table 1 with the heading "Overall" under "System", "Process 1", and " Process 2".

The concept of the BCC model for the one-stage system can be applied to calculate the scale efficiency. Since the system has two processes, there are two scale efficiencies, one for each process. Note that the reason for separating the CCR efficiency into technical and scale efficiencies is to identify the sources of inefficiency so that appropriate amendments can be made to improve performance. To improve efficiency, one can either reduce the input levels while maintaining the same output levels or increase the output levels while consuming the same amount of inputs for pro-