European Journal of Operational Research 210 (2011) 1–9

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Invited Review

The stochastic economic lot scheduling problem: A survey

E.M.M. Winands [a,*], I.J.B.F. Adan [b], G.J. van Houtum [c]

[a] Department of Mathematics, VU University, 1081 HV Amsterdam, The Netherlands
[b] Department of Mathematics and Computer Science, Eindhoven University of Technology, P.O. Box 513, 5600 MB Eindhoven, The Netherlands
[c] Department of Technology Management, Eindhoven University of Technology, P.O. Box 513, 5600 MB Eindhoven, The Netherlands

ARTICLE INFO

Article history:
Received 11 April 2009
Accepted 10 June 2010
Available online 18 June 2010

Keywords:
Inventory theory
Production systems
Stochastic economic lot scheduling problem

ABSTRACT

The present literature survey focuses on the *stochastic economic lot scheduling problem* (SELSP). The SELSP deals with the make-to-stock production of multiple standardized products on a single machine with limited capacity under *random demands*, *possibly random setup times* and *possibly random production times*. The main task of a production manager in this setting is the construction of a production plan for the machine. Based on the critical elements of such a production plan, we present a classification and extensive overview of the research on the SELSP together with an indication of open research areas. By doing so, we intend to stimulate the discussion on the important problems concerning the SELSP both from a theoretical and a practical point of view.

1. Introduction

The present literature review focuses on the *stochastic economic lot scheduling problem*, abbreviated by SELSP. Our main goal is to develop a new framework to classify research on the SELSP and, based on this framework, to point out challenging directions for future research. The SELSP deals with the make-to-stock production of multiple standardized products on a single machine with limited capacity under *random demands*, *possibly random setup times* and *possibly random production times*. The SELSP is a common problem in practice, e.g., in glass and paper production, injection molding, metal stamping and semi-continuous chemical processes, but also in bulk production of consumer products such as detergents and beers. Some specific applications described in the open literature are a laminate manufacturing plant (see [6]), a glass-containers manufacturing company (see [23]), a large consumer products manufacturer (see [26]), a producer of plastic bumpers for cars (see [28]) and an aerospace component supplier (see [47]). The first author has been involved in several industry projects concerning the SELSP, among which two projects at a chemical company (see [58,49]), a plant in an oleochemical production environment (see [42]) and a producer of building materials (see [52]).

In the SELSP, a production policy is needed which describes for each possible state of the system whether to continue production of the current product, whether to switch to another product or whether to idle the machine. The primary goal of such a production policy is the optimization of a pre-defined performance measure. Common performance measures in practice are, e.g., missed demand or service levels, the average work in-progress (WIP) level, the minimization of expected total costs (e.g., setup, inventory and production costs), the fraction of time that is lost due to setups, idle time of the machine, the average stock level or the waiting time of customers.

The development of such a policy for the SELSP is generally regarded as a challenging problem; the finite production capacity has to be *dynamically* distributed among the products in order to be reactive to the stochastic demands, processing and setup times (see [48]). The presence of *setup times* in combination with the *stochastic environment* are the key *complicating factors* of the problem. On the one hand, one aims for short cycle lengths, and thus frequent production opportunities for the various products, in order to be able to react to the stochasticity in the system. On the other hand, short cycle lengths increase the setup frequency, which has a negative influence on the amount of capacity available for production. Consequently, this effect hinders the timely fulfillment of demand.

The aim of the present paper is twofold. First of all, it gives an overview of the research on the SELSP along with a comprehensive list of references. Secondly, several areas for future research of the SELSP are outlined. By doing so, we intend to stimulate the discussion on the important problems concerning the SELSP both from a theoretical and a practical point of view. Therefore, the present survey is directed at both academicians and practitioners. In 1999, a survey on the SELSP by Sox et al. [48] appeared. The motivation for writing a new survey paper is threefold. Firstly, since the publication of the paper by Sox et al. [48], a large number of papers on the SELSP has appeared in the open literature. Secondly, we make a different classification of the literature. That is, we classify the papers

* Corresponding author. Tel.: +31 648696721.
E-mail addresses: emm.winands@few.vu.nl (E.M.M. Winands), i.j.b.f.adan@tue.nl (I.J.B.F. Adan), g.j.v.houtum@tue.nl (G.J. van Houtum).

0377-2217/$ - see front matter © 2010 Elsevier B.V. All rights reserved.
doi:10.1016/j.ejor.2010.06.011

based on the critical elements of the production plan seen by practitioners and not based on the modeling methods introduced by academicians. Thirdly, we give an extensive list of topics for further research both from a theoretical and applied point of view.

The rest of the paper is organized as follows. In Section 2 we give a detailed problem description of the SELSP after positioning the problem in the classical field of multi-product single machine scheduling problems. Furthermore, we describe the relation between the SELSP and its deterministic counterpart. In Section 3 the most important elements of the production plan are discussed. Based on these elements, Section 4 gives an extensive overview of the strategies and the methods studied in the open literature. Section 5 summarizes the most important conclusions drawn from this overview and indicates the gaps in the current state of the research on the SELSP. Finally, the paper is ended by the conclusions in Section 6.

2. Problem statement

The present section is divided into three parts. First, we position the SELSP in the framework of multi-product single machine scheduling problems and, subsequently, describe the relationship between the SELSP and its deterministic counterpart. Finally, we present a detailed problem description.

Scheduling production of multiple products on a single machine under tight capacity constraints is one of the classic problems in operations research. There are many variations of multi-product single machine scheduling problems, but we may classify them by the following three characteristics:

1. *Presence or absence of setup times and/or costs*: The most important impact of setups on the production plan is that the products need to be produced in batches, since otherwise costly capacity is wasted on setups. Therefore, setup times make it impossible to be completely responsive to the demand since switching implies loss of production capacity, as argued by Bourland [7].
2. *Customized or standardized products*: Since customized products can only be produced when there is a request for an order, these products have to be produced in a *make-to-order* fashion. In case of standardized products one may choose a *make-to-stock* production policy, because such products do not have to be produced to customer specifications. It is obvious that standardized products, thus, give more freedom in deciding when to make which product and in what quantity.
3. *Stochastic or deterministic environment*: In a completely deterministic environment one can confine oneself to a rigid production plan which is repeated over and over again. However, when the company has to be responsive to a stochastic environment such a rigid schedule will not suffice anymore.

The Venn diagram in Fig. 1 depicts the eight subproblems, which are created by the above characterization. Of course, this diagram only provides a global classification of multi-product single machine scheduling problems. One can easily think of examples of hybrid systems (see, e.g., [1,21,35] for the combination of make-to-order and make-to-stock). Moreover, besides situations with production capacity constraints, one can also easily imagine cases with a limitation on the aggregate amount of inventory (see, e.g., [39]).

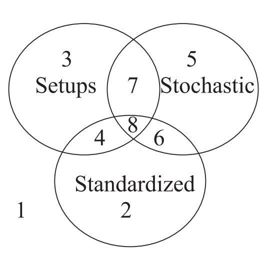

Fig. 1. Multi-product single-machine scheduling problems.

As introduced earlier, the present research considers the production of multiple standardized products on a single machine with limited capacity and setup costs under random demands, possibly random setup times and possibly random production times, depicted as subproblem 8 in Fig. 1: the SELSP. The present paper does not comprise the other subproblems shown in Fig. 1. Suggested books for readers who would like to pursue their knowledge of these subproblems are Silver et al. [44] and Zipkin [60].

2.1. Relation with the deterministic ELSP

The current subsection describes the relationship between the SELSP and subproblem 4, which is usually called the (deterministic) *economic lot scheduling problem* (ELSP). This relation is described for a couple of reasons. First of all, subproblem 4 is closely related to the SELSP, i.e., it can be seen as its deterministic counterpart. Secondly, in developing a production strategy in a stochastic environment the solutions for the ELSP are frequently used as basis for the stochastic solution. Thirdly, in practice a deterministic solution is often wrongly implemented, without any adaptations, in a stochastic environment.

The ELSP has received lots of attention in the literature over the past decades (for surveys see, e.g., [16,43]). In the literature two approaches can be distinguished for the ELSP (see again, e.g., [16]), which has been proven to be NP-hard (see [29]): exact approaches with optimal solutions for restricted problems and heuristic approaches with good solutions for the general problem. Both approaches derive a rigid cyclic schedule, which will be strictly followed until the end of the planning horizon, as described by Gascon et al. [26]. Unfortunately, the solutions of the ELSP can only be applied in an ideal plant, where machines are perfectly reliable, setup and production rates are constant, raw material and tools are always available, demand is known and initial inventories are on level, as argued by Gallego [24]. This is a utopia and, thus, the deterministic problem has to be extended to a stochastic version, the SELSP. We think that it is appropriate that we also bring the paper of Bradley and Conway [9] to the attention, which gives a tutorial on multi-item single machine *deterministic* scheduling problems in which the difficulties arising in stochastic problems are indicated.

Two major differences can be seen between production plans for a deterministic environment on the one hand and a stochastic environment on the other hand. Firstly, a rigid cyclic production plan will not suffice anymore in a stochastic environment, since one has to be responsive to the dynamic changes in this environment. This means that dynamics (e.g., a dynamic production sequence or a dynamic lot-sizing policy) have to be included in the production plan. Secondly, in a stochastic environment the inventory levels for the individual products play a more important role than in the deterministic case, as indicated by Sox et al. [48]. Inventory now does not only reduce the number of setups in a cycle, but it also serve as hedge against stock-outs and scheduling conflicts due to the variation in demand, production or setup times. The inventory levels needed to guarantee a certain service level increase drastically in a stochastic environment. Irrespective of these differences, Bourland [7] rightly notes that the insights and results obtained from the deterministic studies remain useful, since a deterministic production plan is frequently used as a basis for the solution of the stochastic problem.