European Journal of Operational Research 210 (2011) 10–26

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Discrete Optimization

Survivable network design with demand uncertainty

S.E. Terblanche [a,*], R. Wessäly [b,c], J.M. Hattingh [d]

[a] Centre for Business Mathematics and Informatics, North-West University, South Africa
[b] Zuse Institute Berlin, Germany
[c] atesio GmbH Berlin, Germany
[d] School of Computer, Statistical and Mathematical Sciences, North-West University, South Africa

ARTICLE INFO

Article history:
Received 30 November 2009
Accepted 29 September 2010
Available online 7 October 2010

Keywords:
Integer programming
Stochastic programming
Robust optimization
Telecommunication

ABSTRACT

The objective in designing a communications network is to find the most cost efficient network design that specifies hardware devices to be installed, the type of transmission links to be installed, and the routing strategy to be followed. In this paper algorithmic ideas are presented for improving tractability in solving the survivable network design problem by taking into account uncertainty in the traffic requirements. Strategies for improving separation of metric inequalities are presented and an iterative approach for obtaining solutions, that significantly reduces computing times, is introduced. Computational results are provided based on data collected from an operational network.

1. Introduction

Survivability within the context of communication networks can be defined broadly as the property of a network that will allow it to restore traffic, or limit the loss thereof, in the event of component failures. Different network modelling approaches that incorporate survivability requirements have been suggested in the literature. For uncapacitated network design problems (see, e.g. Grötschel and Monma [1], and Stoer [2]) survivability requirements are expressed in terms of the connectivity between communicating node pairs. That is, without taking into account required capacity and the routing that will take place on the network later during its operation, the topology of the network is designed such that specific node or edge connectivity requirements are satisfied. An example of a node (or an edge) connectivity requirement for a communicating node pair is that the network should contain at least a predetermined number of node-disjoint (or edge-disjoint) paths connecting the said node pair. In this paper, however, we are interested in modelling approaches that are aimed at addressing both capacity and routing as part of the network design problem. Therefore, the survivability models applicable in this paper are set within the context of multicommodity flow network design problems (also referred to as capacitated network design problems). The following references provide a summary of selected survivability models that are relevant to our discussion.

The model proposed by Minoux [3] takes failure states into account such that, apart from providing capacity for traffic flow during normal operating conditions, enough capacity is provided to allow re-routing of all traffic in the event of a single component failure. A similar model suggested by Wu [4] allows the re-routing of traffic affected by failures, but without disturbing the flow of traffic in normal operating conditions. The implication of applying the latter is that, in practice, less effort is required to do the re-routing. However, much more computing effort is needed to obtain good solutions. The model suggested by Dahl and Stoer [5] achieves survivability by diversifying flows for each communicating node pair among several paths such that only a prespecified percentage of traffic is affected in the case of network failures. Although this approach cannot ensure complete survivability, it is more favourable from a practical perspective since no re-routing needs to be performed to restore traffic during network failures. For a more detailed discussion of the above-mentioned survivability models, the reader is referred to Wessäly [6]. The survivability model adopted in this paper is a more generalised version of the model suggested by Dahl and Stoer [5].

Apart from dealing with survivability, the need for robust network designs has made it necessary to take uncertainty into account as part of the network design process. Optimisation under uncertainty is nothing new and has been around from as early as the 1950s [7]. The application thereof, however, has only recently become an active point of discussion. The main reason might be that many deterministic optimisation problems, specifically combinatorial problems, are already difficult to solve. Therefore, much of the effort has gone into developing theories and algorithms for solving the "easier" deterministic problem. With the advances

* Corresponding author. Tel.: +27 18 299 2594.
E-mail addresses: fanie.terblanche@nwu.ac.za (S.E. Terblanche), wessaely@zib.de, wessaely@atesio.de (R. Wessäly), giel.hattingh@nwu.ac.za (J.M. Hattingh).

0377-2217/$ - see front matter © 2010 Elsevier B.V. All rights reserved.
doi:10.1016/j.ejor.2010.09.041

made in the field of combinatorial optimisation and new technologies, solving problems with elements of uncertainty has become more attainable. Furthermore, the improvements made towards data storage and data management have also made it easier to obtain large volumes of historical data, making uncertainty modelling more feasible.

In the literature there are mainly two ways of dealing with demand uncertainty within the network design context. Firstly, if something is known about the distribution properties of the demand requirements, a stochastic programming approach can be followed. Otherwise, if such information is not available, then a polyhedral uncertainty model can be applied where demand requirements are modelled as a polyhedron of uncertainty. The latter approach is known in literature as robust optimisation.

1.1. Application of stochastic programming to network design

A chance constrained programming problem has been suggested by Dempster et al. [8] for simultaneously allocating capacity to virtual paths within an ATM network and assigning feasible flows to the virtual paths. The capacity and flow variables are continuous and both the set of demand constraints and capacity constraints are treated as probabilistic. The authors provided an equivalent deterministic linear programming formulation for the problem and presented results for a network problem with 30 nodes, 70 edges and nearly 300 pairs of traffic demands.

A two stage stochastic programming formulation has been suggested by Lisser et al. [9] for finding the optimal capacity assignment by taking into account penalty costs for demand requirements not met. The capacity and flow variables are continuous and as a solution approach an analytic centre cutting plane method is used. Computational results are provided for 16 test instances containing up to 26 nodes and 53 edges. A maximum of up to 80 demand pairs have been considered and the number of scenarios range between 2^8 and 3^7.

In the paper by Riss and Andersen [10] a stochastic integer programming model is considered for the problem of network design with uncertain demands. For this model modular capacities are considered, i.e., capacities can be installed in integer multiples of a low bandwidth type and a high bandwidth type. A bifurcated continuous flow routing model is considered. The solution approach applied is a modified L-shape method that combines Benders decomposition with a branch-and-cut scheme. Proofs are provided to generalise well known valid inequalities, such as metric and partition inequalities, to the case where multiple scenarios for modelling random demands are considered.

1.2. Application of robust optimisation to network design

In the paper by Altm et al. [11], a compact formulation is provided for the Virtual Private Network design problem under traffic uncertainty. The traffic demand requirements are specified as a traffic polytope and details are provided for a column generation and cutting plane algorithm.

Atamtürk and Zhang [12] describe a two stage robust optimisation approach for solving the network design problem with demand uncertainty. Compared to the usual setup of having capacity variables as first stage and flow variables as second stage, the approach followed by Atamtürk and Zhang [12] is to partition the graph associated with the problem into first stage and second stage links. The result of having done this is that a set of capacity variables and flow variables is jointly part of the first stage variables, and another set of capacity variables and flow variables is jointly part of the second stage variables. This setup is useful for capacity expansion type of applications. Theoretical results are provided with regard to the computational complexity of the approach and computational results are presented comparing solutions from two stage robust optimisation with solutions from two stage stochastic programming.

A robust approach for solving the capacity expansion problem with uncertain demand is described by Ordóñez and Zhao [13]. An adjustable robust model is suggested along with conditions making the problem more tractable. The model only caters for continuous capacity and flow variables, and computational results they present are based on a 21-node network with 45 edges.

1.3. Choosing a modelling approach

The problem being considered in this paper is solving the survivable network design problem taking into account a set of non-simultaneous traffic demand vectors. The philosophy is that variation in the network traffic that brings about uncertainty can be approximated by a finite set of non-simultaneous traffic demand vectors. Even though no assumptions are made about the distributional properties of the demand vectors, the problem is approached within the stochastic programming framework. Each demand vector in our set of non-simultaneous demand vectors is considered to be a realisation of a random event with probability of one. However, the true power of stochastic programming will not be exploited since the objective function of the model considered in this paper has no revenue part that will drive the second stage flow variables. Consequently, demand requirements can only be met by implementing hard constraints. It is for this reason that many practitioners prefer to label the problem under investigation in this paper as the multi-hour network design problem. It should be noted that although the main approach followed here is not based on a robust optimisation framework, a robust implementation was used to achieve tractability for a specific aspect of the problem being addressed. The remainder of this section is devoted to literature related to the multi-hour network design problem.

In the paper by Medhi [14] the multi-hour network design problem for reconfigurable ATM networks is considered. The model assumes unsplittable flows by considering a single virtual path (VP) between demands. The concept of different traffic classes is introduced which allows more than one VP per demand pair. Path selection variables are binary and are indexed according to a traffic class, a commodity, and a time period. A modular capacity model is considered since integer variables are used to define the number of capacity units that can be installed on an edge. A Lagrangian decomposition approach is followed by applying subgradient optimisation. In the book by Pioro and Medhi [15] variations of the multi-hour network design problem are presented that include link blocking for circuit-switched networks, reconfigurable splittable flows, and non-reconfigurable unsplittable flows.

The mixed-integer programming model presented by Dutta [16] uses an explicit capacity model where binary variables are used for modelling different capacity types that can be installed on an edge. Integer variables are used for specifying the number of circuits carried on one or more paths in order to satisfy demand requirements. A Lagrangian based heuristic is used as a solution approach. In the paper by Chari and Dutta [17] a similar model is presented, but with the modification that a modular capacity model is considered. That is, the capacity variables are treated as pure integers and as a result the number of units installable for one type of capacity is modelled instead of different capacity types. A Benders decomposition approach is followed whereby both the master and subproblems are solved heuristically.

The model suggested by Amiri and Pirkul [18] is a non-linear mixed integer programming model, where the non-linearity is a result of the requirement for the demand to be modelled as independent M/M/1 queues in which links are treated as servers with service rates proportional to the edge capacities. Binary variables