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Discrete Optimization

The discrete facility location problem with balanced allocation of customers

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ABSTRACT

We consider a discrete facility location problem where the difference between the maximum and minimum number of customers allocated to every plant has to be balanced. Two different Integer Programming formulations are built, and several families of valid inequalities for these formulations are developed. Preprocessing techniques which allow to reduce the size of the largest formulation, based on the upper bound obtained by means of an ad hoc heuristic solution, are also incorporated. Since the number of available valid inequalities for this formulation is exponential, a branch-and-cut algorithm is designed where the most violated inequalities are separated at every node of the branching tree. Both formulations, with and without the improvements, are tested in a computational framework in order to discriminate the most promising solution methods. Difficult instances with up to 50 potential plants and 100 customers, and largest easy instances, can be solved in one CPU hour.

1. Introduction

During the last two decades different models which capture more features of real location problems have been introduced in the literature. Among these features the issue of equity in cost or distance-to-travel distribution, i.e., equity with respect to the set of customers, has attracted considerable attention. Thus for instance Erkut [1] proposed a general framework for quantifying inequality and presented some axioms for the appropriateness of the inequality measures. Berman and Kaplan [2] addressed the equity question using taxes while Drezner et al. [3] dealt with the minimization of the Gini index when a facility is located. The interested reader can consult Marsh and Schilling [4] and Eiselt and Laporte [5], two reviews of the literature on equity measurement in Location Theory. General equitable objective functions are studied in the recent work of Marín et al. [6], where the formulation developed in Marín et al. [7] for the Discrete Ordered Median Problem is extended and improved. A different approach, in which a discrete facility location problem with a form of equity criterion called customers' envy is introduced, can be consulted in Espejo et al. [8]. Another related paper is Berger and Bechwati [9], where the authors maximize customer equity when organizing promotion budget allocation.

The papers cited above study models that look for equitable solutions with respect to the distance traveled by the customers. Just a handful of papers take into account equity from the point of view of the providers. One of these papers is Berman et al. [10]. These authors adopt the point of view of the providers, in the sense that an equitable solution is such that the amount of load assigned to each provider is balanced. Concretely, they consider the problem of locating p facilities to serve a set of customers with given demands, such that the maximum demand attracted to each facility is minimized. Another paper that deals with providers equity is Kalcsics et al. [11]. These authors consider several ordered location problems. Among them, the *ordered capacitated facility location problem from the supplier point of view*, where the supplier cost of a facility is the sum of the transportation costs of shipments from the facility to the customers, is studied. The objective is built sorting the supplier costs and multiplying them by a weights vector (plus a term associated to the setup costs). Adequately choosing these weights, different equitable objectives can be managed.

To the best of our knowledge, there is not any other equitable –with respect to the providers point of view – discrete location model in the literature. To partially fill this gap, this paper examines a discrete location problem that consists of establishing a fixed number of p plants to cover n demand points in such a way that every customer is allocated to its closest plant and the number of customers allocated to each plant is balanced. Instead of considering a *minimax* objective, we try to increase the balance among the suppliers using a *range* objective, i.e., minimizing the difference between the supplier with maximum number of assigned customers and the supplier with

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minimum number of assigned customers. The reader should note that without the constraint of closest allocation the problem lacks interest since any set of p locations gives lots of optimal solutions.

Some applications of this model can be found in the field of *Territory Design* (see [12]), where small geographic areas must be grouped into larger geographic clusters according to some planning criteria (for instance, political districting, solid waste collection, school districting). Another application is location of antennas for mobile phones. In general, our model will be useful when absolute distances between customers and suppliers can be neglected but either customers will not accept other provider than the closest one – schools, voting stations – or customers must be allocated to its closest provider for technical reasons – the case of antennas –. That is to say, only relative distances are meaningful in the model. The reader may note that using ordinal ranking in location problems is not new and there is an important body of literature in competitive location that deals with this issue (see e.g. [13–16]).

The paper is organized as follows. In Section 2 we formally state the problem and give some notation. The problem is modeled as an Integer Programming Problem in Section 3, where two different formulations are introduced. Valid inequalities for both formulations are derived in Section 4. These inequalities, together with preprocessing strategies, formulations strengthening and heuristic solutions make up the improvement phase to which Section 5 is devoted. The resulting solution algorithms are detailed in Section 6 and tested in Section 7. Finally, some conclusions and future research lines are presented in Section 8.

2. Problem description

Let $A = \{1,\ldots,n\}$ be a set used to represent *customers* and let $B = \{1,\ldots,m\}$ be a set used to represent *potential plants*. Let $C = (c_{ij})_{i\in A, j\in B}$ be a matrix of *costs* (transportation costs, distances). We do not impose any additional constraint on the matrix C (costs can be negative, do not need to satisfy triangle inequality nor symmetry). A solution to the *Balanced Location Problem* (LOBA) is given by a set of plants $X \subset B$, $|X| = p$, such that each customer is allocated to its closest plant, i.e., given a solution X, we assume that each customer $i \in A$ will be supplied from a plant $j \in X$ such that $c_{ij} = \min_{k\in X}\{c_{ik}\}$. In case of tie, the customer can be assigned to any of his/her closest plants. The objective is to minimize the difference between the maximum and the minimum number of customers allocated to any plant in X.

Example 1. Consider $\{0,3,4,10\}$ four points in the real line corresponding with both the set of customers and the set of potential plants. Let c_{ij} be the Euclidean distance between customer i and potential plant j. Let $p = 2$ be the number of plants to be located.

In this example there are six possible choices for X, represented in the six lines of Table 1. Five of the six choices result in three customers allocated to one of the plants and one customer allocated to the other plant, giving an objective value of $3 - 1 = 2$. The remaining choice, (2,3), gives the optimal value 0, since both plants receive two customers and the solution is perfectly balanced.

Example 1 gives an objective value of 0 because a solution exists where the same amount of customers is allocated to each plant. We give now another example where such a perfect allocation does not exist.

Example 2. Consider the case where $n = m = 3$ and the cost matrix is

$$C = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}.$$

Regardless the number of plants p, all customers will be allocated to the same plant, the one with the minimum index. If $p \geqslant 2$, one plant will receive all the customers and the rest of plants will not receive any customer. Therefore, the optimal value to LOBA will be 3.

Example 2 can be easily extended to any number of customers and potential plants to show that the optimal solution to LOBA can be extremely unbalanced.

3. Integer Programming formulations

In the following, we give two Integer Programming formulations for LOBA. This first one is called *standard* since it comes from the standard formulation for the p-median problem adding two variables (maximum and minimum number of customers allocated to any plant) and the closest allocation constraints. The second formulation is called *ordered* because it shares the main structure with some formulations recently developed for the Discrete Ordered Median Problem, among others (see [6]).

Table 1
Example of LOBA.

X	Allocation				# Allocated	Objective
(1,2)	1	2	2	2	(1,3)	2
(1,3)	1	3	3	3	(1,3)	2
(1,4)	1	1	1	4	(3,1)	2
(2,3)	2	2	3	3	(2,2)	0
(2,4)	2	2	2	4	(3,1)	2
(3,4)	3	3	3	4	(3,1)	2