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New stochastic models for preventive maintenance and maintenance optimization

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ABSTRACT

This paper considers periodic preventive maintenance policies for a deteriorating repairable system. On each failure the system is repaired and, at the planned times, it is periodically maintained to improve its reliability performance. Most of periodic preventive maintenance (PM) models for repairable systems have been studied assuming that the failure process between two PMs follows the nonhomogeneous Poisson process (NHPP), implying the minimal repair on each failure. However, in this paper, we assume that the failure process between two PMs follows a new counting process which is a generalized version of the NHPP. We develop two types of PM models and study detailed properties of the optimal policies which minimize the long-run expected cost rates. Numerical examples are also provided.

1. Introduction

In the reliability area, various types of repairs have been developed based on different point processes and they have been applied for modeling the corresponding maintenance effects. After the classical maintenance model proposed by Barlow and Hunter (1960), mathematical sophistication of preventive maintenance models has increased. Comprehensive discussions on different maintenance models can be found in Canfield (1986), Nakagawa (1986), Nguyen and Murthy (1981) and Liu et al. (1995). More theoretical and sophisticated models have also been developed; see Aven and Jensen (2000), Cheng and Chen (2003), Marais and Saleh (2009), Mercier (2002), Mercier and Pham (2012), Vaughan (2005), Wang and Zhang (2013) and Chenetal (2015). See also Valdez-Flores and Feldman (1989), Wang (2002) and Tadj, Ouali, Yacout, and Ait-Kadi (2011) for surveys on various practical maintenance models. Nakagawa (2005) provides an overview on more theoretical maintenance models.

Maintenance actions can generally be divided into two types: corrective maintenance (CM) and preventive maintenance (PM). For a deteriorating repairable system, the CM action is conducted upon failure to recover the system from a failure, whereas the PM action is performed at the planned time to improve the system reliability performance. Most of the periodic PM models for repairable systems have been studied assuming that the failure process between two PMs follows the nonhomogeneous Poisson process (NHPP), which implies that the repair type of the CM performed on each failure is the minimal repair (see, e.g., Cheng & Chen, 2003; Cheng, Zhao, Chen, & Sun, 2014; Nakagawa, 1986; Nguyen & Murthy, 1981; Park, Jung, & Yum, 2000). By the 'minimal repair', we mean that the state of the system after the repair is restored to the as-bad-as-old condition, i.e., to the state it had prior to the failure. As the failure process in this case follows the NHPP, the assumption of minimal repair generally allows a closed-form of results (e.g. the long-run expected cost rates) and nice mathematical properties for the optimal solutions in maintenance optimization.

However, practically, when a component in a system fails, this may lead to a more hostile working environment through increased pressure, temperature, humidity, and so on, causing instantaneous stress or damage to the adjacent non-failed components. It eventually results in the system degradation and, hence, an increase in the level of the system failure rate function (see El-Damcese, 1997; Hoyland & Rausand, 1994; and Jeong, 2012). For example, (a) the failure of a still wire cable in a bridge or in an elevator instantaneously increases the stress on the remaining cables and leads to some damages before repairing the failed one; (b) for a multi-engine airplane, the failure of an engine during flight instantaneously causes increased stress on the non-failed engines until landing for the repair of the failed engine; (c) a failure of a pump in a multi pump hydraulic control system instantly increases the pressure for each non-failed pump until the repair of the failed one; (d) when an electric device fails by an external shock (electric or mechanical shock), the non-failed components also experience this external shock and their reliability performances can be worse than before.

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Under the situations illustrated above, after the repair of the failed component, the reliability performances of the non-failed components are worse than before the failure. Accordingly, the overall state of the system after the repair of the failed component is worse than the state it had just prior to the failure. As illustrated above, in practice, this type of 'worse-than-minimal repair' happens not because of a repair action (e.g., a faulty repair), but because of the negative effect caused by a failure in the system.

Motivated by the above consideration, in this paper, we introduce a new repair type which is 'worse-than-minimal repair', based on the Generalized Polya Process (GPP). The GPP has been recently described based on the notion of stochastic intensity and its probabilistic properties have been studied in detail (Cha, 2014). It will be seen that this new repair process includes the minimal repair process as a special case and, accordingly, it is a generalized version of the minimal repair process. It will also be seen that the GPP relaxes the restrictive 'independent increments property' of the NHPP, which has an important practical meaning from maintenance modeling point of view (see Section 2). Then, assuming that the failure process between two PMs follows the GPP, we study and discuss a relevant maintenance optimization problem.

The main objective of Cha (2014) was to study the probabilistic properties of the GPP. Then, it suggested a possible application of the GPP to a reliability problem by providing an example which applies the GPP to the simplest 'replacement' policy. On the other hand, in this current paper, we develop two types of PM models based on the new repair process, which combine CM, PM and replacement. In the two PM models, the system is preventively maintained (PM) at periodic times iT, $i = 1, 2, \ldots, N-1$, and is replaced at NT (Replacement). The reliability improvement made by a PM is stochastically modeled based on the properties of the GPP. The failures of the system which occur between PMs are repaired by the new type of repair (CM). Thus, the corresponding PM policies are characterized by two PM parameters (N, T). For each model, we study detailed properties of the optimal PM policies (N^*, T^*) which minimize the long-run expected cost rates. Because the PM models in this paper are developed based on a new type of repair process instead of the conventional minimal repair process, the current work would suggest a new research direction in the study of PM.

This paper is organized as follows. In Section 2, we introduce a new type of generalized repair which is worse-than-minimal-repair based on the GPP. We also interpret the involved parameter in the new repair process from the PM modeling point of view. In Section 3, we develop the two types of PM models. For each model, we derive the properties of the optimal policy and, based on them, we provide two-stage optimization procedure. In Section 4, numerical examples are provided for illustrations. Finally in Section 5, concluding remarks are given and further topics to be developed are suggested.

2. A generalized repair process

2.1. A new type of repair

Although the minimal repair process based on the NHPP has been a very useful tool for modeling the failure process of a repairable system, its practical limitation also exists. For instance, the NHPP possesses the independent increment property. This implies that the future failure process in this case does not depend on the failure history of the system. For instance, suppose that we have two systems at time t: one has experienced no failure until time t, whereas the other one has experienced frequent failures and has been minimally repaired until time t. Under the minimal repair assumption, these two systems have no statistical difference at all, e.g., their future failure rates are the same. On the other hand, as will be seen in this section, the GPP does not possess the independent increment property and, under the GPP repair assumption, the future reliability performance of a system becomes worse and worse as the number of system failures occurred in the past increases. Thus, from this practical point of view, the GPP repair can be understood as a practically more plausible assumption in maintenance modeling. Some recent works such as Babykina and Couallier (2014) and Le Gat (2014) have shown that the GPP is better fitted to some real field failure data sets than the traditional NHPP model.

Note that one counting process corresponds to one repair type and vice versa. For instance, the 'perfect repair' corresponds to the renewal process and the 'minimal repair' corresponds to the NHPP. Thus, to define a new type of repair, we need a new counting process. We will now introduce the concept of stochastic intensity and, based on it, we provide the definition of the GPP.

Let $\{N(t),\ t \geq 0\}$ be an *orderly* point process and $H_{t-} \equiv \{N(u),\ 0 \leq u < t\}$ be the history (internal filtration) of the process in $[0, t)$, i.e., the set of all point events in $[0, t)$. Observe that H_{t-} can equivalently be defined in terms of $N(t-)$ and the sequential arrival points of the events $S_0 \equiv 0 \leq S_1 \leq S_2 \leq \ldots \leq S_{N(t-)} < t$ in $[0, t)$, where S_i is the time from 0 until the arrival of the ith event in $[0, t)$. The point processes can be mathematically described by using the concept of the stochastic intensity (the intensity process) $\lambda_t,\ t \geq 0$ (Aven & Jensen, 1999, 2000). As discussed in Cha and Finkelstein (2011), Finkelstein and Cha (2013) and Cha (2014), the stochastic intensity λ_t of an orderly point process $\{N(t),\ t \geq 0\}$ is defined as the following limit:

$$\lambda_t = \lim_{\Delta t \to 0} \frac{\Pr[N(t, t+\Delta t) = 1 | H_{t-}]}{\Delta t} = \lim_{\Delta t \to 0} \frac{E[N(t, t+\Delta t) | H_{t-}]}{\Delta t}, \tag{1}$$

where $N(t_1, t_2)$, $t_1 < t_2$, represents the number of events in $[t_1, t_2)$. The stochastic intensity defined in (1) has the following heuristic interpretation: $\lambda_t dt = E[dN(t)|H_{t-}]$, which is very similar to the ordinary failure rate or hazard rate of a random variable (Aven & Jensen, 1999). In the case of the NHPP with intensity function $\lambda(t)$, the stochastic intensity is given by the 'deterministic function' $\lambda_t = \lambda(t),\ t \geq 0$. In Cha (2014), the definition of the GPP is given as follows.

Definition 1. Generalized Polya Process (GPP)

A counting process $\{N(t),\ t \geq 0\}$ is called the Generalized Polya Process (GPP) with the set of parameters $(\lambda(t), \alpha, \beta)$, $\alpha \geq 0$, $\beta > 0$, if

(i) $N(0) = 0$;
(ii) $\lambda_t = (\alpha N(t-) + \beta)\lambda(t)$.

As mentioned in Cha (2014), the GPP with $(\lambda(t), \alpha = 0, \beta = 1)$ reduces to the NHPP with the intensity function $\lambda(t)$ and, accordingly, the GPP can be understood as a generalized version of the NHPP. For our further discussion, we give here the following supplementary results on the GPP. The proofs are given in Cha (2014).

Proposition 1. *Suppose that a counting process $\{N(t),\ t \geq 0\}$ is the GPP with the set of parameters $(\lambda(t), \alpha, \beta)$, $\alpha \geq 0$, $\beta > 0$. Then*

$$P(N(t) = n) = \frac{\Gamma(\beta/\alpha + n)}{\Gamma(\beta/\alpha) n!} (1 - \exp\{-\alpha \Lambda(t)\})^n \times (\exp\{-\alpha \Lambda(t)\})^{\frac{\beta}{\alpha}}, \quad n = 0, 1, 2, \ldots,$$

where $\Lambda(t) \equiv \int_0^t \lambda(s) ds$, and

$$E[N(t)] = \frac{\beta}{\alpha} (\exp\{\alpha \Lambda(t)\} - 1).$$

Thus, it can be seen that the distribution of $N(t)$ follows a negative binomial distribution with the corresponding parameters $(\exp\{-\alpha \Lambda(t)\}, \beta/\alpha)$, where the probability mass function of