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Decision Support

A new bid price approach to dynamic resource allocation in network revenue management

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ABSTRACT

Firms selling perishable products use a variety of techniques to maximize revenue through the dynamic control of their inventories. One of the most powerful and simple approaches to address this issue consists of assigning threshold values (“bid prices”) to each resource, and to accept requests whenever their revenue exceeds the sum of the bid prices associated with its constituent resources. In this context, we propose a new customer choice-based mathematical program to estimate time-dependent bid prices. In contrast with most approaches from the current literature, ours is characterized by its flexibility. Indeed, it can easily embed technical and practical constraints that occur in most central reservation systems (CRS). To solve the model, we develop a column generation algorithm, in which the NP-hard subproblem is addressed via an efficient heuristic procedure. Our computational results illustrate the performance of the method, through comparisons with alternative proposals.

1. Introduction

In network revenue management, capacity control involves the design of rules that specify whether individual requests for products should be accepted or not, taking into account that products use resources that are in limited supply. Since the aim is to maximize revenue by controlling resource availability over a prespecified booking horizon, this issue is the core of the optimization process. A popular means of addressing it is through ‘bid prices’, which were introduced by Simpson (1989) and Williamson (1992). According to this policy, a request for an available product is accepted if and only if the revenue exceeds the sum threshold values (bid prices) associated with the product’s constituent resources (Talluri & Van Ryzin, 2005).

Among the many approaches to optimal resource allocation that have been proposed in this context, some take explicitly into account the choice behavior of rational customers. This is the case of the deterministic linear programming formulation CDLP introduced by (Bront, Méndez-Díaz, & Vulcano, 2009). However, due in part to the computational complexity of solving CDLPs of practical sizes, most Central Reservation Systems (CRS) implement bid prices or set booking limits[1] on the number of products that can be accessed (Meissner & Strauss, 2012a).

In the airline industry, bid price controls have become the method of choice for seat inventory control problem for other reasons as well. First, since in each period a single value (bid price) is assigned to each resource — the number of which is generally far fewer than the number of products in most real-world network settings — the number of decision parameters is greatly reduced. Next, the decision-making process can be implemented quickly and very simply. Indeed, whenever a request arrives, one only needs to compare the revenue to the sum of the corresponding bid prices. Finally, the concept of bid price control is intuitive and easy to understand. Even if the approach cannot theoretically guarantee the optimal revenue, good bid prices can yet lead to a significant revenue increase. In some cases, asymptotic optimality can even be proved under weak assumptions (Talluri & Van Ryzin, 2005).

Talluri and van Ryzin (1998) provided the theoretical foundations for the bid price approach. In particular, they extended the initial approximation of bid price values as the opportunity cost of one additional capacity unit by Simpson (1989) and Williamson (1992). They specified bid prices for each resource, each time

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[1] In the airline industry, this refers to policies that set bounds on fare products. Bid price policies can be interpreted as ‘dual’ methods that achieve a similar goal.

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period, each capacity. They also provided a two-period counterexample that showed that bid prices do not necessarily yield an optimal control policy. More recently, (Topaloglu, 2009) showed how to compute bid prices that depend on residual resource capacities, through the Lagrangian relaxation of certain capacity constraints.

In the context of choice based network revenue management, (Chaneton & Vulcano, 2011) proposed a bid price policy for addressing a continuous capacity/demand model. The model allows a simple calculation of the revenue function's sample path gradient, which is then embedded within a stochastic steepest ascent algorithm that converges towards a stationary point of the revenue function. In the static case, (Chaneton, Méndez-Díaz, Vulcano, & Zabala, 2010) developed a framework for solving the CDLP linear program, by focusing on offer sets that are compatible with at least one bid price policy. This feature is shared by our model, where the compatibility condition explicitly enters the column generation framework. In a recent work, (Meissner & Strauss, 2012a) describes a heuristic that iteratively improves an initial guess of bid prices. Those bid prices could be provided by a dynamic estimate of the capacities' marginal values.

Based on the customer choice-based deterministic linear programming paradigm, this paper's main contribution is the design of a procedure for determining time-dependent bid prices that is numerically efficient and achieves strong revenue performance. The model's structure enables to consider the topology (hub-and-spoke) of the network, the business rules specific to the industry, as well as the behavior of customers, who base their purchase decisions upon the products' attributes of the products and their willingness to pay. In order to solve realistic instances, we develop a column generation algorithm that is based on an efficient heuristic procedure for solving the NP-hard subproblem, in the spirit of Chaneton et al. (2010). In parallel, we introduce two filtering approaches that reduce the size of the problem. These are compatible with arbitrary control policies and allow the exact solution of small instances by off-the-shelf solvers.

The rest of the paper is organized as follows. Section 2 is devoted to the formulation of the inventory management model, i.e., a bid price model for choice-based network revenue management. Solution algorithms are presented in Section 3. In Section 4, we provide computational results, as well as comparisons with alternative approaches from the recent literature. Finally, concluding comments and avenues for further research are outlined in Section 5.

2. Problem formulation

In this section, we introduce the general definitions and notation, and provide mathematical formulations of the bid price model.

2.1. General definitions and notation

Let us consider a set of products $j \in J = \{1, 2, \ldots, |J|\}$, composed of resources $i \in I = \{1, 2, \ldots, |I|\}$. The use of resources $i \in I = \{1, 2, \ldots, |I|\}$ by the products is specified by an incidence matrix A of dimension $|I| \times |J|$, whose binary elements are defined as

$$a_{ij} = \begin{cases} 1, & \text{if resource } i \text{ is used by product } j, \\ 0, & \text{otherwise.} \end{cases}$$

We associate with each product a revenue (fare) r_j, and with each resource a capacity c_i. We denote by r (respectively c) the fare (respectively capacity) vector.

To account for the heterogeneity of the population, customers are partitioned into segments indexed by $l \in L = \{1, 2, \ldots, |L|\}$ and is characterized by attributes: time, price, path preferences, etc. The probability that a customer belong to segment l is given by p_l. We associate with each segment l the consideration set Γ_l, which specifies the subset of products considered by a customer belonging to segment l. The utility of product j to a member of segment l is denoted by v_{lj}.

Time $t \in T = \{1, 2, \ldots, |T|\}$ runs forward in discrete increments. Within each time period t, the firm must decide which subset $S \subseteq J$ of products is made available to customers. The binary variable $\xi_j(S)$ is then set to 1 if product j belongs to set S, and to 0 otherwise.

The arrival process of customers is governed by independent Poisson processes of respective rates $\lambda_l = \lambda p_l$. The total arrival rate is then $\lambda = \sum_{l=1}^{|L|} \lambda_l$.

Let $P_j(S)$ denote the probability that a customer select product j, and let $P_0(S) = 1 - \sum_{j \in S} P_j(S)$ be the residual probability associated with the no-purchase option. Assuming that customers' choice probabilities are dictated by a discrete choice model, the probability that customer l select an available product $j \in S$ is given by

$$P_{lj}(S) = \frac{v_{lj}}{v_{l0} + \sum_{h \in \Gamma_l \cap S} v_{lh}} \tag{1}$$

$$= \frac{\xi_j(S) v_{lj}}{v_{l0} + \sum_{h \in J} \xi_h(S) v_{lh}} \tag{2}$$

This definition is compatible with the multinomial logit choice model, where v_{lj} is the value assigned to product j by a customer from segment l. We set $v_{lj} = 0$ if $\notin \Gamma_l$.

Since the firm does not have prior knowledge of the segment associated with an arriving customer, the probability that a product j be selected is given by

$$P_j(S) = \sum_{l=1}^{|L|} p_l P_{lj}(S). \tag{3}$$

For a specific offer set S, the expected revenue is expressed as

$$R(S) = \sum_{j \in S} r_j P_j(S). \tag{4}$$

Now, let $Q_i(S)$ denote the probability of using a unit of resource $i \in I$, regrouped into the vector $Q(S)$. Letting $P(S) = (P_1(S), \ldots, P_n(S))^\top$, we can write

$$Q(S) = AP(S). \tag{5}$$

Based on this notation, a *bid price policy* assigns to each resource i a *bid price* π_i. According to a policy $\pi = (\pi_i)_{i \in I}$, a product $j \in S$ is *offered* at time t, i.e., j belongs to S, if and only if

$$r_j \geq \sum_{i \in I} a_{ij} \pi_{it}. \tag{6}$$

2.2. Model formulation I

For each time index $t \in T$, let us denote by ξ_{jt} the binary variable that specifies whether product j belongs to the set S at time t, and by ξ the vector obtained by concatenation of these variables. The bid price policy that achieves the highest return is than an optimal solution for the mathematical program:

$$\text{BID-Ia:} \quad \max_{\xi, \pi} \sum_{t \in T} \sum_{j \in J} \sum_{l \in L} \lambda p_l r_j \frac{\xi_{jt} v_{lj}}{v_{l0} + \sum_{h \in J} \xi_{ht} v_{lh}}, \tag{7}$$

subject to

$$\sum_{t \in T} \sum_{j \in J} \lambda p_l a_{ij} \frac{\xi_{jt} v_{lj}}{v_{l0} + \sum_{h \in J} \xi_{ht} v_{lh}} \leq c_i \quad \forall\, i \in I, \tag{8}$$

$$r_j > \sum_{i \in I} a_{ij} \pi_{it} \quad \Leftrightarrow \quad \xi_{jt} = 1 \quad \forall\, t \in T, \forall\, j \in J, \tag{9}$$