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Decision Support

A branch-and-Benders-cut method for nonlinear power design in green wireless local area networks

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ABSTRACT

We consider a problem arising in the design of green wireless local area networks. Decisions on powering-on a set of access points (APs), via the assignment of one power level (PL) to each opened AP, and decisions on the assignment of the user terminals (UTs) to the opened APs, have to be taken simultaneously. The PL assigned to an AP affects, in a nonlinear way, the capacity of the connections between the AP and the UTs that are assigned to it. The objective is to minimize the overall power consumption of the APs, which has two components: location/capacity dimensioning costs of the APs; assignment costs that depend on the total demands assigned to the APs. We develop a branch-and-Benders-cut (BBC) method where, in a non-standard fashion, the master problem includes the variables of the Benders subproblem, but relaxes their integrality. The BBC method has been tested on a large set of instances, and compared to a Benders decomposition algorithm on a subset of instances without assignment costs, where the two approaches can be compared. The computational results show the superiority of BBC in terms of solution quality, scalability and robustness.

1. Introduction

We address an optimization problem arising in the design of green (or energy-saving) wireless local area networks (WLANs). We focus on the design of efficient reconfiguration algorithms to reduce the power consumption of the WLAN infrastructure when the load is scarce. Most of the currently deployed enterprise WLANs are continuously operated at full power, i.e., all access points are always turned on with the transmission power set to the maximum. This produces a considerable waste of energy, because the same power is employed at the peak hours and during the off peak periods. We address this issue by proposing an optimization model that is used to take two kinds of decisions: (i) associate each user with one of the available access points and (ii) set the transmission power level of each access point.

The area of wireless network design requires the development of optimization models and methods (see Kennington, Olinick, & Rajan, 2010 for an overview of the main challenges in this field). Recent contributions include the development of a nested Benders decomposition method (Naoum-Sawaya & Elhedhli, 2010) that combines classical Benders decomposition (Benders, 1962) with combinatorial Benders decomposition (Codato & Fischetti, 2006); and the derivation of pure 0–1 programming formulations, tightened with strong valid inequalities (D'Andreagiovanni, Mannino, & Sassano, 2013) (based on the Ph.D. Thesis of the first author D'Andreagiovanni, 2012).

The problem we consider is defined on a bipartite network structure, with a set of access points (APs) that must be assigned to user terminals (UTs) in order to satisfy the user demands, without exceeding the capacity of the connections between the APs and the UTs. Each UT must be assigned to exactly one powered-on AP. Several different power levels (PLs) are available for powering on each AP. If an AP is powered-on, then exactly one PL must be associated with it.

A key issue arises concerning the capacity of the connections between the APs and the UTs: the specific PL assigned to a (powered-on) AP affects, in a nonlinear way, the capacity of the connections between the AP and the UTs assigned to it. The only assumption is that the transmission capacity between a UT and an AP is a nonnegative nondecreasing function of the radiated power at the AP, which will be formally defined in Section 2. As a result, the optimization model is an integer

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nonlinear program, a class of notoriously difficult mathematical programs.

The objective is to minimize the overall power consumption of the APs, which has two components. The first component includes the location and capacity dimensioning costs of the APs, i.e., the costs associated with powering-on APs and assigning a PL to each of them. The second component concerns the assignment costs between UTs and APs, which are given by a linear dependency between the power consumed by the APs and the total demands assigned to the APs. In an earlier contribution by the same authors (Gendron, Garroppo, Nencioni, Scutellà, & Tavanti, 2013), it was assumed that the power consumed by an AP does not depend on the demands assigned to that AP and, therefore, only the first component of the objective function was considered. The presence of the second component, called *UT assignment costs*, yields a more realistic problem formulation that also complicates the development of a solution method, as discussed below.

We propose to address the intrinsic difficulties of the problem, i.e., nonlinear capacity constraints and complex objective function, by developing an exact algorithm inspired by Benders decomposition (Benders, 1962). Since the Benders subproblem in our approach is a 0–1 program and not a linear program (LP), we use *canonical cuts* (Balas & Jeroslow, 1972), as in logic-based Benders decomposition (Hooker & Ottosson, 2003) and combinatorial Benders decomposition (Codato & Fischetti, 2006), instead of the classical LP duality-based Benders cuts. The resulting Benders cuts are improved by simple arguments (also used in (Gendron et al., 2013)) based on the assumption that the transmission capacity functions are nondecreasing.

In a non-standard fashion, our master problem includes the variables of the Benders subproblem, but relaxes their integrality. This feature provides a simple, yet efficient, way to consider the UT assignment costs in the master problem to ensure that effective lower bounds are computed. Linear approximations of the nonlinear transmission capacity functions are also included in the formulation of the master problem. As a result, the master problem is a mixed-integer linear programming (MILP) relaxation, which we solve with a state-of-the-art MILP software tool (a similar approach that solves a nonlinear integer program through the addition of Benders cuts in a MILP relaxation can be found in (Zhang, Romero, Beck, & Amon, 2013)). Instead of solving one MILP master problem at every iteration, we use a *branch-and-Benders-cut (BBC) method*, also called Benders-based branch-and-cut method, where a single branch-and-cut (B&C) tree is constructed and the Benders cuts are added during the exploration of the B&C tree.

In the constraint programming (CP) literature, this algorithmic scheme is known as *branch-and-check* (Thorsteinsson, 2001) and has been the object of empirical work comparing it to a more traditional iterative Benders decomposition approach (Beck, 2010). In these CP references, the Benders subproblem is solved with CP and the Benders cuts are based on the notion of inference dual, introduced in logic-based Benders decomposition (Hooker & Ottosson, 2003). In the operations research (OR) literature, BBC has attracted the attention of many researchers recently, as it makes better use of the reoptimization capabilities of the MILP solvers than the classical Benders decomposition approach. This is discussed, for instance, in (Naoum-Sawaya & Elhedhli, 2013), which uses an interior-point method to solve a Benders reformulation in a BBC framework, applying it to facility location and network design problems. Other recent implementations of the BBC method include: (Fortz & Poss, 2009), which compares BBC to classical Benders decomposition for a multi-layer network design problem, showing significant speedups on average; (de Camargo, de Miranda Jr., & Ferreira, 2011), which combines the generation of outer approximation and Benders cuts in a BBC method for the single allocation hub location problem under congestion; (Botton, Fortz, Gouveia, & Poss, 2013), where a BBC method is used to solve a hop-constrained survivable network design problem; (Adulyasak, Cordeau, & Jans, 2013), which uses BBC algorithms for solving production routing problems under demand uncertainty. In all these OR references, the Benders subproblem is an LP and the Benders cuts are based on LP duality, as in the approach originally proposed by Benders (1962). As mentioned above, our Benders subproblem is a 0–1 program and we make use instead of canonical cuts (Balas & Jeroslow, 1972). Canonical cuts in a wireless network design problem (different from ours) have also been used in (Naoum-Sawaya & Elhedhli, 2010). A major difference between existing contributions and our paper is that our master problem includes the variables of the Benders subproblem, but relaxes their integrality. In the above references, a traditional partitioning of the variables into master problem variables and subproblem variables is used. Note that this traditional partitioning has been questioned recently in the context of stochastic programming (Crainic, Hewitt, & Rei, 2014), where a new approach called partial decomposition was proposed, in which a subset of scenario subproblems are kept in the master problem.

This paper is a follow-up on an earlier contribution by the same authors (Gendron et al., 2013) on the special case without UT assignment costs, for which a Benders decomposition method has been proposed. This method corresponds to a cutting-plane approach where feasibility cuts are iteratively added to the master problem, thanks to the information provided when solving the Benders subproblem. The latter is a feasibility problem, because of the absence of UT assignment costs. This is in contrast with the Benders subproblem defined in the present paper, which is an optimization problem, given the inclusion of UT assignment costs. This is a major difference, as the presence of such additional assignment costs prevents a straightforward extension of the Benders decomposition approach used in (Gendron et al., 2013), as we clarify in Section 3.5. Another notable difference is that the master problem in (Gendron et al., 2013) does not include the variables of the Benders subproblem. The decomposition adopted in (Gendron et al., 2013) thus follows a traditional variable partitioning approach as in the original Benders method (Benders, 1962), where the variables of the master problem and those of the subproblem do not overlap. In Section 4, we compare the performance of the two methods on the special case without UT assignment costs addressed in (Gendron et al., 2013). The computational results show the superiority of the proposed BBC approach in terms of solution quality, scalability and robustness.

The paper is organized as follows. In Section 2, we describe the problem, which we denote as GWLANP, and we present the integer nonlinear programming model we propose for the GWLANP. The BBC method is described in Section 3. Computational results from experiments on randomly generated realistic instances are reported in Section 4. The conclusion summarizes our findings and identifies promising research directions.

2. Problem description and formulation

In order to state the GWLANP in a formal way, we need to characterize the energy consumed by the powered-on APs and the capacity of the connections between the APs and the UTs. First, let us denote by $\mathcal{I}$, $\mathcal{J}$ and $\mathcal{K}$ the sets of UTs, APs and PLs, respectively.

Concerning the energy consumed by the powered-on APs, the power consumed by $j \in \mathcal{J}$ is composed of a fixed component and of two variable components. The fixed component, denoted p_0, is bound to the mere fact that the device is powered-on, and therefore, it encompasses AC/DC conversion, basic circuitry powering, dispersion, etc. The first variable power component associated with $j \in \mathcal{J}$ is given by its radiated power π_j, which depends on the PL assigned to $j \in \mathcal{J}$. More precisely, if $k \in \mathcal{K}$ is assigned