European Journal of Operational Research 205 (2010) 325–331

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Production, Manufacturing and Logistics

A search method for optimal control of a flow shop system of traditional machines

Omer Selvi, Kagan Gokbayrak *

Department of Industrial Engineering, Bilkent University, Ankara 06800, Turkey

ARTICLE INFO

Article history:
Received 18 July 2008
Accepted 26 December 2009
Available online 11 January 2010

Keywords:
Search method
Controllable service times
Convex programming
Trust-region methods
Flow shop

ABSTRACT

We consider a convex and nondifferentiable optimization problem for deterministic flow shop systems in which the arrival times of the jobs are known and jobs are processed in the order they arrive. The decision variables are the service times that are to be set only once before processing the first job, and cannot be altered between processes. The cost objective is the sum of regular costs on job completion times and service costs inversely proportional to the controllable service times. A finite set of subproblems, which can be solved by trust-region methods, are defined and their solutions are related to the optimal solution of the optimization problem under consideration. Exploiting these relationships, we introduce a two-phase search method which converges in a finite number of iterations. A numerical study is held to demonstrate the solution performance of the search method compared to a subgradient method proposed in earlier work.

1. Introduction

We consider flow shop systems consisting of traditional human operated (non-CNC) machines that are processing identical jobs. During mass production, a company cannot afford human interventions to modify the service times because the setup times for these modifications are idle times for these machines. Moreover, these manual modifications are prone to errors. Therefore, we assume that the service times at these traditional machines are initially controllable, i.e., they are set at the start-up time, and are applied to all jobs processed at these machines.

The cost to be minimized is assumed to consist of service costs on machines, which are dependent on service times, and regular completion time costs for jobs, e.g., inventory holding costs. Motivated by the extended Taylor's tool-wear equation (see in Kalpakjian and Schmid (2006)), we assume that faster services increase wear and tear on the machine tools due to increased temperatures and may raise the need for extra supervision, increasing service costs. The degradation of the product quality due to faster services are also lumped into these service costs. Slower services, on the other hand, build up inventory and postpone the completion times increasing the regular completion time costs. Our objective in this study is to determine the cost minimizing service times.

The scheduling problems of flow shops are known to be NP-hard even for fixed service times (see in Pinedo (2002)). In these problems, the objective is to find the best sequence of jobs to be processed at machines. Except for two-machine systems with the objective of minimizing makespan, the scheduling literature is limited to heuristics and approximate solution methods. Introduction of controllable service times at machines further complicates the problem. Following the pioneering work in Vickson (1980), controllable service times have received great attention over the last three decades. Nowicki and Zdrzalka (1988) studied a two-machine flow shop system with the objective of minimizing a cost formed of makespan and decreasing linear service costs. Through reducing the knapsack problem to it, the problem was proven to be NP-hard even in the case where the service times are controllable only at the first machine. The heuristic algorithm proposed in this work was later extended to flow shops with more than two machines in Nowicki (1993). In Cheng and Shakhlevich (1999), an algorithm for a similar cost structure was presented for proportionate permutation flow shops where each job is associated with a single service time for all machines. Karabati and Kouvelis (1997) addressed the problem of minimizing a cost formed of decreasing linear service and regular cycle time costs, and introduced an iterative solution procedure where the task of selecting the optimal service times for a given sequence was formulated as a linear programming problem solved by a row generation scheme. Furthermore, a genetic algorithm for large problems was presented whose effectiveness was demonstrated through numerical studies. A survey of results on the controllable service times can be found in Nowicki and Zdrzalka (1990), Hoogeveen (2005) and Shabtay and Steiner (2007).

The studies above assumed the service costs to be decreasing linear functions of service times. This linearity assumption, however, fails to reflect the *law of diminishing marginal returns*:

* Corresponding author. Tel.: +90 312 290 3343.
E-mail addresses: selvi@bilkent.edu.tr (O. Selvi), kgokbayr@bilkent.edu.tr (K. Gokbayrak).

0377-2217/$ - see front matter
doi:10.1016/j.ejor.2009.12.028

productivity increases at a decreasing rate with the amount of resource employed. Therefore, in this study, we adapt the service cost function $\theta_j(\cdot)$ on machine j defined as

$$\theta_j(s_j) = \frac{\beta_j}{s_j^{\alpha}}, \tag{1}$$

where β_j is a positive parameter, s_j is the service time at machine j, and α is a positive constant. This cost structure was shown to correspond to many industrial operations in Monma et al. (1990). Such nonlinear and convex service costs were also considered in Gurel and Akturk (2007).

The optimal control literature assumed that jobs are served in a given sequence, and concentrated on determining the optimal control inputs which in turn determine the optimal service times. Pepyne and Cassandras (1998) formulated a nonconvex and nondifferentiable optimal control problem for a single machine system with the objective of completing jobs as fast as possible with the least amount of control effort. The results were extended in Pepyne and Cassandras (2000) for jobs with completion due dates and a cost structure penalizing both earliness and tardiness. In Cassandras et al. (2001), the task of solving these problems was simplified by exploiting structural properties of the optimal sample path. Further exploiting the structural properties of the optimal sample path, "backward in time" and "forward in time" algorithms based on the decomposition of the original nonconvex and nondifferentiable optimization problem into a set of smaller convex optimization problems with linear constraints were presented in Wardi et al. (2001) and Cho et al. (2001), respectively. The "forward in time" algorithm presented in Cho et al. (2001) was then improved in Zhang and Cassandras (2002). Mao et al. (2004) removed the completion time costs and introduced due date constraints. Some optimal solution properties of the resulting problem were identified leading to a highly efficient solution algorithm.

The work on two-machine systems started out with Cassandras et al. (1999), which derived some necessary conditions for optimality and introduced a solution technique using the Bezier approximation method. Extending the work in Mao et al. (2004), Mao and Cassandras (2006) considered a two-machine flow shop system with service costs that are decreasing on service times, and derived some optimality properties that led to an iterative algorithm, which was shown to converge. Gokbayrak and Selvi (2006) studied a two-machine flow shop system with a regular cost on completion times and decreasing costs on service times, and identified some optimal sample path characteristics to simplify the problem. In particular, no waiting was observed between machines on the optimal sample path leading to the transformation of the nonconvex discrete-event optimal control problem into a simple convex programming problem. Gokbayrak and Selvi (2007) extended the no-wait property to multimachine flow shop systems. Using this property, simpler equivalent convex programming formulations were presented and "forward in time" solution algorithms were developed under strict convexity assumptions on service and completion time costs. Gokbayrak and Selvi (2010) generalized the results to multimachine mixed line flow shop systems with Computer Numerical Control (CNC) and traditional machines. The no-wait property was shown to exist for the downstream of the first controllable (CNC) machine of the system. Employing this result, a simplified convex optimization problem along with a "forward in time" decomposition algorithm were introduced enabling for solving large systems in short times and with low memory requirements.

Employing the cost structure in Gokbayrak and Selvi (2007), Gokbayrak and Selvi (2008) and Gokbayrak and Selvi (2009) considered a deterministic flow shop system where the service times at machines are set only once, and cannot be altered between processes. Gokbayrak and Selvi (2008) derived a set of waiting characteristics in such systems and presented an equivalent simple convex optimization problem employing these characteristics. In order to eliminate the need for convex programming solvers, Gokbayrak and Selvi (2009) derived additional waiting characteristics and introduced a minmax problem, which is almost everywhere differentiable, of a finite set of convex functions along with its subgradient descent solution algorithm. In this study, we propose an alternative solution method for the minmax problem in Gokbayrak and Selvi (2009). The relationships between the minimizers of the convex functions in the minmax problem and the optimal solution are derived. These relationships suggest a two-phase search algorithm that determines the optimal solution in a finite number of iterations. In each iteration a convex optimization problem needs to be solved. For the special case where the service cost structure is as in (1) allowing us to sort the service times of the machines, these convex optimization problems are solved by trust-region methods.

The rest of the paper is organized as follows: In Section 2, we describe the problem and present the minmax formulation given in Gokbayrak and Selvi (2009). In Section 3, we derive the relationships between the optimal solution and the minimizers of the convex functions in the minmax formulation. Consequently, the two-phase search algorithm is presented in this section. Implementation details of this search algorithm are given in Section 4 for the service cost structure in (1). Section 5 demonstrates the solution performance of the proposed methodology by a numerical study. Finally, Section 6 concludes the paper.

2. Problem formulations

Let us consider an M-machine flow shop system with unlimited buffer spaces between machines. A sequence of N identical jobs, denoted by $\{C_i\}_{i=1}^{N}$, arrive at this system at known times $0 \leqslant a_1 \leqslant a_2 \leqslant \cdots \leqslant a_N$. Machines process one job at a time on a first-come first-served non-preemptive basis, i.e., a job in service can not be interrupted until its service is completed. The service time at each machine j, denoted by s_j, is the same for all jobs and is the jth entry of the service time vector $s = (s_1, \ldots, s_M)$.

We consider the discrete-event optimal control problem P, which has the following form:

$$P : \min \left\{ J(s) = \sum_{j=1}^{M} \theta_j(s_j) + \sum_{i=1}^{N} \phi_i(x_{i,M}) \right\} \tag{2}$$

subject to

$$x_{i,j} = \max(x_{i,j-1}, x_{i-1,j}) + s_j \tag{3}$$

$$x_{i,0} = a_i, \quad x_{0,j} = -\infty \tag{4}$$

$$s_j \geqslant 0 \tag{5}$$

for $i = 1, \ldots, N$ and $j = 1, \ldots, M$, where $x_{i,j}$ denotes the departure time of job C_i from machine j.

In this formulation, θ_j denotes the service cost at machine j and ϕ_i denotes the completion time cost for job C_i. The following assumptions are necessary to make the problem somewhat more tractable while preserving the originality of the problem.

Assumption 1. $\theta_j(\cdot)$, for $j = 1, \ldots, M$, is continuously differentiable, monotonically decreasing and strictly convex.

Assumption 2. $\phi_i(\cdot)$, for $i = 1, \ldots, N$, is continuously differentiable, monotonically increasing and convex.

Note that for the costs satisfying these assumptions, longer services will decrease the service costs, while increasing the completion times, hence the completion time costs. This trade-off is what makes our problem interesting.