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Decision Support

A parallel multiple reference point approach for multi-objective optimization

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ABSTRACT

This paper presents a multiple reference point approach for multi-objective optimization problems of discrete and combinatorial nature. When approximating the Pareto Frontier, multiple reference points can be used instead of traditional techniques. These multiple reference points can easily be implemented in a parallel algorithmic framework. The reference points can be uniformly distributed within a region that covers the Pareto Frontier. An evolutionary algorithm is based on an achievement scalarizing function that does not impose any restrictions with respect to the location of the reference points in the objective space. Computational experiments are performed on a bi-objective flow-shop scheduling problem. Results, quality measures as well as a statistical analysis are reported in the paper.

1. Introduction

Multi-objective optimization (MO) is one of most challenging areas in the field of Multiple Criteria Decision Analysis (MCDA). Over the last decades, a large number of papers were published in this field comprising both theoretical and applied works. The most challenging problems in MO are related to the identification of the Pareto Frontier (PF), or an approximation of it (PF^A) for large-size and rather difficult MO problems. For such a purpose, evolutionary algorithms seem more adequate than exact methods. However, the identification of the whole set or of an approximation of the PF is frequently not necessary, an approximation of some specific regions suffices. Indeed, when some preference information is provided by the Decision-Maker (DM), diverse methods can guide, in an interactive manner, the search of the potentially best compromise solution(s) (which is an efficient solution) in a particular region of interest. Reference point methods are particularly adequate to deal with this kind of situations; the preference information needed by them has mainly the form of reference point(s) (or also any other information that can be translated into reference point(s)). Reference point-based methods use then an achievement scalarizing function to make projection of the reference points onto the PF.

Contrary to the single-objective case, typically there is no unique optimal solution for a MO problem. Instead, a set of solutions, called Pareto solutions, efficient solutions, etc., represent the PF when transformed into the objective space. A fundamental issue while trying to solve MO problems is related to the cooperation between the DM and a computerized Decision Support System (DSS). In general, the DSS includes a mathematical model of the problem being solved along with a data base, an optimization solver, and an interactive solution procedure. There are several approaches to the roles that the DM could play in a decision-making process. Firstly, the *a priori* approaches, where the DM is supposed to provide some knowledge or preferences about the problem to be solved in order to help the DSS in its search; practical experience shows that such methods are seldom effective. Separately, the *a posteriori* approaches, where the DSS aims at finding, or approximating the whole set of efficient solutions; the DM then has to choose his/her most preferred one. Finally, in interactive approaches, there is a progressive direct interaction/cooperation between the DM and the DSS.

Over the last two decades, most of MO resolution methods proposed in the literature were rather the *a posteriori* ones. A large part of them consist of approximating the set of efficient solutions and the corresponding PF using an evolutionary algorithm. On the one hand, this is based on the belief that the computing power of modern computers is unlimited, we can use them for any complex problems and solution methods. This belief, however, is contradicted by computational experience of solving complex problems: even the most powerful computer of any generation can be easily saturated, due to the non-linear dependencies of

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computational complexity on the amount of data processed. Thus, a reasonable use of the existing computing power, even if this power is tremendous due to the possibilities of parallel use of computers in the network, still remains and will probably always remain a fundamental problem. On the other hand, many interactive approaches are based on the reference point method using achievement scalarizing functions as proposed by Wierzbicki (1980) and developed by many other researchers; see, for example, Wierzbicki et al. (2000). The reference point method results in projecting a given reference point (or a pair of them, usually called reservation and aspiration points), that represents the objective, criteria or outcome values desired by the DM, onto the set of efficient solutions. There are diverse interaction protocols within the framework of reference point approaches, starting with just fully sovereign change of reference points by the DM, through using additional trade-off information, up to visual interfaces based on fuzzy specification of reference values. However, the result is focusing on a specific region of the objective space, thus avoiding the loss of computational resources for searching solutions that may not interest the DM at the end.

In this paper, we propose a new method combining the use of reference points while trying to approximate the whole set of efficient solutions, or a selected part of it. Instead of using a single-reference point, the idea of this *a posteriori* approach is to automatically define a set of points in such a way that the objective space is uniformly divided, but entirely covered. Each point gives rise to a corresponding achievement scalarizing function that concentrates on a specific sub-region of the objective space. Thus, the set of efficient solutions can be rebuilt by combining the output of all solvers. Notice that the solvers can be launched in parallel since the problems of optimizing a given achievement function can all be solved independently. The proposed parallel multiple reference point approach can be used to solve difficult real-world optimization problems, and it is here applied to a bi-objective combinatorial scheduling problem.

The outline of the paper is as follows. Section 2 introduces some fundamental concepts related to MO. Section 3 is devoted to the multiple reference point approach proposed in the paper; the key issues being widely detailed. Section 4 presents the parallel model and the implementation of the method. Section 5 formulates a bi-objective flow-shop scheduling problem (FSP). Section 6 shows the effectiveness of our approach by conducting experiments on the FSP. Finally, the last section concludes the paper and draws some perspectives.

2. Multi-objective optimization (MO)

Many areas of the industry as, for example, telecommunications, transportation, aeronautics, chemistry, mechanical, and environment, deal with MO, where various conflicting objectives have to be considered simultaneously. This section briefly presents some basic concepts, definitions, and notation for MO. The interested reader is referred to Miettinen (1999), Deb (2001), and Coello Coello et al. (2002) for more details about this field.

2.1. Basic concepts

A general MO problem consists of optimizing a set of $n \geqslant 2$ objective functions $f_1(x), f_2(x), \ldots, f_n(x)$. Each objective function can be either minimized or maximized; or even stabilized, kept close to a given target level (Wierzbicki et al., 2000). Here we assume, without loss of generality, that all are to be minimized. A decision vector $x = (x_1, x_2, \ldots, x_k)$ is represented by a vector of k decision variables. Let X denote the set of *feasible solutions* in the *decision space* $\mathbb{R}_0^k$ $(X \subseteq \mathbb{R}_0^k)$. To each decision vector $x \in X$ is assigned exactly one objective vector, $z \in Z$, on the basis of a vector function $f : X \to Z$ with $z = (z_1, z_2, \ldots, z_n) = f(x) = (f_1(x), f_2(x), \ldots, f_n(x))$, where $Z = f(X)$ denotes the set of feasible points in the *objective* (or *criterion*) *space* $\mathbb{R}^n$ $(Z \subseteq \mathbb{R}^n)$. Therefore, a MO problem can be formulated as follows:

$$\begin{array}{ll} \min & f(x) = (f_1(x), f_2(x), \ldots, f_n(x)) \\ \text{subject to} & x \in X, \end{array} \tag{1}$$

Whereas solving a single-objective optimization problem generally results in a unique optimal solution, a MO problem obtains rather a set of solutions known as *Pareto optimal*. A fundamental concept is the one of *dominance* that can be defined as follows.

Definition 1 (*Dominance*). A solution $x_1 \in X$ *dominates* another solution $x_2 \in X$ if and only if $\forall i \in \{1, \ldots, n\}$, $f_i(x_1) \leqslant f_i(x_2)$ and $\exists j \in \{1, \ldots, n\}$ such that $f_j(x_1) < f_j(x_2)$.

The following two concepts depend on the dominance concept.

Definition 2 (*Efficiency*). A solution $x^* \in X$ is *efficient* if and only if there is not another solution $x \in X$ such that x dominates x^*.

The whole set of efficient solutions is the Pareto optimal set, and is denoted by X_P. The image of a Pareto optimal solution in the objective space results in a *non-dominated outcome vector*.

Definition 3 (*Non-dominated outcome vector*). A point $z \in Z$ is a *non-dominated outcome vector* if there exists at least one efficient solution $x \in X_P$ such that $z = f(x)$.

The set of all non-dominated outcome vectors is the *Pareto Frontier* (PF). One of the possible approaches for solving MO problems consists of finding PF or an approximation PF^A. This depends on the practical computational complexity of the problem, because finding a representation of PF is practically possible only if the resulting computational complexity is rather low.

Now, suppose that the optimum is known for each objective function, then we can define the concept of *ideal vector*:

Definition 4 (*Ideal vector*). The *ideal vector* $z^* = (z_1^*, z_2^*, \ldots, z_n^*)$ is the vector that optimizes each objective function individually

$$z_i^* = \min_{x \in X} f_i(x).$$

Of course, this ideal vector optimizing each objective function is rarely feasible as the objectives are often in conflict. Besides, the upper bounds for all objectives of the PF can be represented by the *nadir point* z^n. This nadir point is much more difficult or impossible to compute (Miettinen, 1999), especially when the number of objectives is more than two. A rough approximation of the nadir point can be provided by recording the maximal values of all objective functions obtained from their separate minimization, while determining the ideal point.

2.2. Achievement scalarizing functions

The *achievement scalarizing function* approach, proposed by Wierzbicki (1980), is frequently used to solve MO problems. This technique is particularly well-suited to work with reference points. A reference point gives desirable or acceptable values for each one of the objective functions. These objective values are called *aspiration levels* and the resulting objective vector is called a *reference point* and can be defined either in the feasible or in the infeasible region of the objective space. One of the families of achievement functions can be stated as follows:

$$\sigma(z, z^0, \lambda, \rho) = \max_{i=1,2,\ldots,n} \{\lambda_i (z_i - z_i^0)\} + \rho \sum_{i=1}^{n} \lambda_i (z_i - z_i^0), \tag{2}$$