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Discrete Optimization

The tournament scheduling problem with absences

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ABSTRACT

We study time scheduling problems with allowed absences as a new kind of graph coloring problem. One may think of a sport tournament where each player (each team) is permitted a certain number t of absences. We then examine how many rounds are needed to schedule the whole tournament in the worst case. This upper limit depends on t and on the structure of the graph G whose edges represent the games that have to be played, but also on whether or not the absences are announced before the tournament starts. Therefore, we actually have two upper limits for the number of required rounds. We have $\chi^t(G)$ for pre-scheduling if all absences are pre-fixed, and we have $\chi^t_{OL}(G)$ for on-line scheduling if we have to stay flexible and deal with absences when they occur. We conjecture that $\chi^t(G) = \Delta(G) + 2t$ and that $\chi^t_{OL}(G) = \chi'(G) + 2t$. The first conjecture is stronger than the Total Coloring Conjecture while the second is weaker than the On-Line List Edge Coloring Conjecture. Our conjectures hold for all bipartite graphs. For complete graphs, we prove them partially. Lower and upper bounds to $\chi^t(G)$ and $\chi^t_{OL}(G)$ for general multigraphs G are established, too.

1. Introduction

There are many different types of scheduling problems. Some of them arise in pure mathematics, but many emerge directly out of real-life needs. For example, good schedules are needed for the assignment of channels or frequencies in communication networks. They are also needed for the allocation of venues and time slots to the teams in sport competitions. Most of these problems can be studied as graph coloring problems, either edge or vertex coloring problems. The graph coloring rule that adjacent edges (resp. vertices) should receive different colors then reflects the most basic requirement of conflict avoidance, the avoidance of overlapping appointments in timetables. Usually, however, there are additional constraints reflecting additional requirements and wishes. For instance, in sport league scheduling, one wants to avoid that a team plays many consecutive games in its hometown. Simultaneously, one wants to minimize the travel distances of the teams. Moreover, TV networks may want the most attractive games to be scheduled at certain dates. There is an economic interest behind many scheduling requirements. Therefore, scheduling has turned into a research area of its own. Usually, this diverse area is studied in operations research and computer science. There is a vast literature, see e.g. Drexl and Knust. (2007), Kendall, Knust, Ribeiro, and Urrutia (2010), Lewis and Thompson (2011), and Rasmussen and Trick (2008), to mention but a few. On the web page (Knust, 2014), many references on various topics in sports scheduling are classified according to different aspects. For mathematical basics about the theory of graphs and multigraphs (graphs which may have multiple edges between any two vertices), different coloring concepts and notational foundations, the reader may consult Diestel (2010), Fiorini and Wilson (1977), Jensen and Toft (1995), and Yap (1996).

In the present paper, we examine edge colorings of multigraphs with a new kind of constraint related to absences. The underlying research should mainly be of interest for people working in the theory of graph colorings. We hope, however, that our results and conjectures will also attract interest in the sport scheduling community. In fact, our research is motivated by time scheduling problems as they arise in sport tournaments or in the scheduling of timetables at schools. Typically, timetables are set up under the assumption that everything goes fine and all participants are available without absences. In real life, however, things often do not go as planned. People get sick or otherwise indisposed. In this case, the best plans can be thrown over. Therefore, it is important to see how one can deal with absences. Apparently, this problem was not studied in literature yet, at least not in any systematic way. The closest mathematical concepts, so far, were *list edge coloring* (Borodin, Kostochka, & Woodall, 1997; Galvin, 1995; Häggkvist & Janssen, 1997), *on-line list edge coloring* (Schauz, 2008; 2014) and *total coloring* (Yap, 1996). Our results heavily rely on the findings in these fields, as we will see. For the general discussion, however, we need to have mathematical concepts that model time scheduling with absences even closer. In order to find suitable definitions, we

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	Round 1	Round 2	Round 3	Round 4
Player A	C	free	**absent**	B
Player B	free	C	**absent**	A
Player C	A	B	free	**absent**

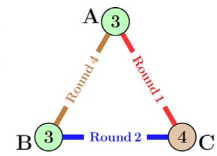

Fig. 1. Optimal schedule for the indicated pre-fixed absences. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

	Round 1	Round 2	Round 3	Round 4
Player A	C	free	**absent**	free
Player B	free	C	free	**absent**
Player C	A	B	free	free

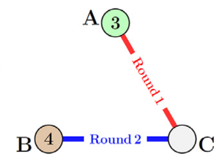

Fig. 2. Worst possible unannounced absences after two completed rounds. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

first need to distinguishing two kinds of absences, *pre-announced pre-fixed absences* and *unannounced absences*. The following example, with its two parts, illustrates the two types of absences and their impact on the number of rounds that is needed to accommodate all games of a tournament. It also explains the graph-theoretic model that we use.

Example 1.1. *Part 1:* Three chess players A, B, C want to play three chess games, the game $A-B$, the game $A-C$ and the game $B-C$. Each player can play at most one game per round. Without absences, this can be done in three rounds. One simply has to play one game per round.

If each player is allowed to miss one round, one round that he has to pre-announce and prefix, then we may need to arrange one additional round. If player A does not come to the first round, player B does not come to the second round and player C does not come to the third round, then three rounds are still enough. However, in all cases in which at least two players miss simultaneously one of the first three rounds, a fourth round has to be arranged. The case where player A and player B cancel the third round and player C cancels the fourth round (whether or not the fourth round needs to take place) is illustrated in the assignment of exponents in Fig. 1. Here, a fourth round can actually not be avoided. For the given absences, four rounds are the minimum, and the presented schedule is optimal in that sense. Next to the time-table, a corresponding graph coloring is also presented. This graph-theoretic model shows players as vertices whose color and number indicate the round in which a player is absent. Games are displayed as edges whose color and number indicate the round in which the game shall take place. One can show that four rounds are always enough. For pre-fixed single absences, four rounds is the upper limit.

Part 2: The situation gets worse if the players do not have to announce their absences in advance and simply do not show up to one round. In this case, it could happen that all players come to the first and second round. After these two rounds there is still at least one game $X-Y$ left over, no matter how the first two rounds are used. So, two players $X, Y \in \{A, B, C\}$ did not play yet. Now, player X may not show up to the third round and player Y may not show up to fourth round. For instance, if the game $A-B$ was not played in the first two rounds ($\{X, Y\} = \{A, B\}$), this could look as in Fig. 2. In this case, a fifth round has to be arranged to accommodate the game $X-Y$. Since at that point, player X and player Y have used up their allowed absences, they actually will attend the fifth round and the tournament can be concluded there. One can show that five rounds are always enough. For unannounced single absences, five rounds is the upper limit.

This example shows that for unannounced absences more rounds might be needed to accommodate all games, compared to the situation with pre-fixed absences. Out of this observation, we address two problems. *Pre-scheduling* with pre-announced pre-fixed absences only, and *on-line scheduling*, where all absences are unannounced and just happen on the fly. We provide upper and lower bounds on the number of rounds that is needed to complete all planned games of a tournament. Of course, this can only be done with some information about the absences. If one team is absent all the time, we never will finish. Therefore, it seems natural to restrict the number of absences by some limit. We may permit each player (team) only a certain number t of absences. We may also permit different players v different numbers $t(v)$ of absences. Apart from that, any choice of matches between the players is allowed. These matches form the edges of a multigraph G. With these notations and parameters, the best general upper bound for the number of rounds is defined as a new kind of chromatic index. We call this index *t-avoiding chromatic index* $\chi^t(G)$, respectively *on-line t-avoiding chromatic index* $\chi^t_{OL}(G)$ – one for pre-fixed and one for unannounced absences. These numbers are the upper limits for the number of rounds in an optimal scheduling. This means, if we know the number $\chi^t(G)$, resp. $\chi^t_{OL}(G)$, then we know the precise number of required rounds if the absences appear as unfortunately as possible, within the given frequency limitations $t(v)$.

There is also a game-theoretic description of our scheduling problem and the numbers $\chi^t(G)$ and $\chi^t_{OL}(G)$. We will not use this approach later on, but we briefly describe it here, as it clarifies things. The whole scheduling process can be seen as a meta-game between two meta-players, an *Organizer* and an *Indisposer*. While Organizer is trying to organize a tournament G within a certain number χ of rounds, Indisposer is trying to prevent that by making the players up to t many times indisposed. There are two versions of that game, one for pre-announced absences and one for unannounced absences. In the first version, Indisposer has only one move, in which he determines all absences. He may enter them into a tabula like the one in Fig. 1. Afterwards, Organizer has to complete the whole schedule in one move by completing the tabula. In the second version, the only difference is that the tabula is filled column by column. Each round, Indisposer indicates absences in one column, and then Organizer completes that column. This could go as in Fig. 2 where $\chi = 4$ columns are not enough to finish the complete tournament $G = K_3$ if $t = 1$ many absences are available in each row. In this game-theoretic setting, the number $\chi^t(G)$, resp. $\chi^t_{OL}(G)$, is the smallest number of columns χ for which a winning strategy for Organizer exists.

We can calculate the numbers $\chi^t(G)$ and $\chi^t_{OL}(G)$ in several cases. In particular, we know $\chi^t(B)$ and $\chi^t_{OL}(B)$ for all bipartite multigraphs B and constant or *blockwise constant* t (with t being