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Innovative Applications of O.R.

Fuzzy multi-period portfolio selection with different investment horizons

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ABSTRACT

This paper considers a fuzzy multi-period portfolio selection problem with V-Shaped transaction cost. Compared with the traditional studies assuming that assets have the same investment horizon, we handle the practical but complicated situation in which assets have different investment horizons. Within the framework of credibility theory, a mean-variance model is formulated with the objective of maximizing the terminal return under the total risk constraint over the whole investment. Alternatively, a variation is given by minimizing the total risk under the terminal return constraint. A fuzzy simulation based genetic algorithm (FSGA) is designed and three numerical examples are given to illustrate the effectiveness of the proposed approach.

1. Introduction

Portfolio selection theory has been rapidly developed since Markowitz (1952) published the seminal work. By quantifying the investment return as expected value and quantifying the investment risk as variance, the optimal portfolio should maximize the expected value of portfolio return under certain variance constraint or minimize the variance of portfolio return under certain expected value constraint. Markowitz's work provides the fundamental principles for modern portfolio selection theory. Several extensions are proposed afterwards by considering transaction costs, trading size and turnover constraints, sensitivity analysis to changes in expected return of the risky assets and other practical requirements (Feinstein & Thapa, 1993; Grinold & Hahn, 2000; Konno & Wijayanayake, 2001; Merton, 1972; Qin, 2015; Soleimani, Golmakani, & Salimi, 2009). For example, Merton (1972) derived the mean-variance portfolio efficient frontiers and verified the characteristics for these frontiers. Soleimani et al. (2009) considered the minimum transaction lots, cardinality constraints and market capitalization.

The mean-variance approach needs to solve a quadratic programming model. For large-scale portfolio selection problems, it has difficulty in finding the optimal solution timely. To overcome this disadvantage, absolute deviation was used to construct a mean-absolute deviation model (Konno & Yamazaki, 1991). Simaan (1997) analyzed the differences thoroughly between the mean-variance model and the mean-absolute deviation model. Liu (2011) conducted two bi-level programming models to calculate the lower and upper bounds of the investment return by using mean-absolute deviation models. In the above works, the investment risk was denoted by the first central absolute moments or the second order central moments of portfolio return, which treat high returns as equally undesirable as low returns. However, since investors care more about the part where the return is lower than the expected value, it is not reasonable to quantify the risk by variance or absolute deviation. Therefore, semivariance was proposed to denote the investment risk by taking the negative part of variance (Markowitz, 1959). Hogan and Warren (1974) defined the semicovariance and developed a mean-semivariance portfolio selection model. Grootveld and Hallerbach (1999) studied the properties and computation problem of mean-semivariance models. Ballestero (2005) applied the Sharpe single index approach to solve the mean-semivariance model. Another well-known downside risk measure is semi-absolute deviation, which was first proposed by Speranza (1993) and extended by Papahristodoulou and Dotzauer (2004). Other risk definitions in portfolio selection include value-at-risk (Linsmeier & Pearson, 2000), conditional value-at-risk (Rockafellar & Uryasev, 2000, 2002), entropy (Kapur & Kesavan, 1992), semi-entropy (Zhou, Li, & Pedrycz, 2016), disutility-based risk measure (Fulga, 2016b), expected shortfall with loss aversion (ESLA) (Fulga, 2016a) and so on.

Except for return and risk, skewness is the third popular objective for researchers and practitioners. Kraus and Litzenberger (1976) pointed out the importance of the third central moment of

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return, which was used to measure the asymmetry degree of return distributions. Skewness was first considered in portfolio selection by Lai (1991). Konno, Shirakawa, and Yamazaki (1993) proposed a mean-absolute deviation-skewness optimization model and applied it to the real-life stock market. Konno and Suzuki (1995) extended the mean-variance model to a mean-variance-skewness model and proposed three computational schemes for solving an associated nonconcave maximization problem.

The above portfolio models characterize the returns of risky assets as random variables with given probability distributions, which is based on the assumption that the future returns can be precisely reflected by the historical data. However, in practice, it is difficult to obtain the precise probability distributions as well as enough data due to the ever-changing economic environment. On the other hand, there are many non-probabilistic factors in real-world portfolio decision-making, including social, political, people's cognitive and psychological factors (Basse, Friedrich, & Vazquez Bea, 2009), etc. Investors often obtain vague information with linguistic descriptions such as high risk, low profit and so on. In this case, fuzzy portfolio models behave better than probabilistic models and it is better to define the returns of risky assets as fuzzy variables (Huang, 2012; Vercher & Bermudez, 2013; Wang & Zhu, 2002). With the introduction of fuzzy set theory, more and more scholars are engaged in portfolio selection studies based on fuzzy set theory. For instance, Carlsson and Fullér (2001) introduced lower and upper possibilistic mean for fuzzy numbers. Zhang and Nie (2003) defined the lower and upper variance and covariance for fuzzy numbers and formulated a fuzzy mean-variance model. Zhang, Zhang, and Xiao (2009) proposed a portfolio selection model with the maximum utility based on the interval-valued possibilistic mean and possibilistic variance. Bhattacharyya and Kar (2011) proposed another mean-variance-skewness model by applying the concept of weighted possibilistic moments of fuzzy numbers. Li, Guo, and Yu (2015) redefined the concepts of mean and variance for fuzzy numbers, proposed a concept of possibilistic skewness, and then formulated a fuzzy mean-variance-skewness portfolio selection model. Although possibility theory is widely used in portfolio selection, there are still some limitations since possibility measure is not self-dual. To overcome this disadvantage, the credibility measure was proposed (Liu & Liu, 2002), and is accepted by more and more researchers (Huang, 2008; Qin, Li, & Ji, 2009; Vercher & Bermudez, 2015; Zhang, Zhang, & Chen, 2011; Zhang, Zhang, & Cai, 2010b). Huang (2008) measured the risk by entropy and presented credibilistic mean-entropy models. Qin et al. (2009) studied the credibilistic cross-entropy minimization portfolio selection model by defining the divergence from investment return to ideal return as fuzzy cross-entropy, average return as expected value and risk as variance, semivariance and chance of bad outcome, respectively. Li, Qin, and Kar (2010) defined the skewness for fuzzy variable within the framework of credibility theory to formulate a fuzzy mean-variance-skewness model. Li, Shou, and Qin (2012) proposed an expected regret minimization model to minimize the expected distance between the maximum return and the investment return. Vercher and Bermudez (2015) proposed a credibilistic mean-absolute semi-deviation portfolio selection model and applied the evolutionary algorithm to find the approximated Pareto frontier.

All the above studies consider single-period investment which makes a one-off decision at the beginning of the period and holds on until the end of the period. However, in practice, investors often need to reallocate their wealth in several consecutive periods for long investment. Hence, it is natural to extend the single-period models to multi-period models. Several research work has been carried out by Hakansson (1971), Li and Ng (2000), Wei and Ye (2007), Fu, Lari-Lavassani, and Li (2010), Wang and Forsyth (2011) and others on multi-period portfolio selection models (Gao, Xiong, & Li, 2016; Yao, Li, & Li, 2016). To our knowledge, there are some research papers related to fuzzy multi-period portfolio selection problem. Liu, Zhang, and Xu (2012) proposed four fuzzy multi-period portfolio optimization models by considering multiple criteria. Zhang, Liu, and Xu (2012) designed a hybrid intelligent algorithm to solve multi-period possibilistic mean-semivariance-entropy model. Liu and Zhang (2015) used a fuzzy decision technique to express investor's preference and formulated a multi-period mean-semivariance model with transaction lots within the framework of possibility theory. There are some limitations for the above portfolio models in real applications. Fu et al. (2010) used the Hamilton–Jacobi–Bellman (HJB) equation to obtain the explicit closed form solutions of efficient frontier. But this method is not more feasible for the massive calculation when there are large amounts of risky assets. Liu et al. (2012) and Liu and Zhang (2015) built the multi-period portfolio models within the framework of possibility theory and assumed that all the risky assets had the same investment horizon, which means that all the returns invested in this period can be obtained when this period ends. Thus, the multi-period portfolio model is just a simple accumulation of multiple single period models. However, in practice, the investment horizons for assets are generally different. The multi-period portfolio selection model becomes complicated when return at each period consists of various parts. Although some risky assets are invested at different periods, they may reach their maturity dates at the same period. For this unorthodox portfolio selection model, traditional mathematical programming methods are generally inefficient on finding the optimal solutions due to the complexity and massive calculation. The contribution of this paper can be stated as follows:

- To address the investor's asset allocation for different investment horizon with several periods.
- To design a fuzzy simulation based genetic algorithm (FSGA) for approximate optimal solution.

The rest of the paper is organized as follows. Section 2 reviews the preliminaries about fuzzy variables and credibility theory. Section 3 gives explicit expressions for returns at different periods by analyzing their compositions, and presents two multi-period mean-variance models. Section 4 introduces fuzzy simulation-based genetic algorithm. Section 5 presents two numerical examples to demonstrate the effectiveness of our proposed algorithm. Section 6 concludes the paper.

2. Preliminaries

To have a better understanding on this paper, some fundamental concepts of credibility theory are introduced in this section. Let Θ be a nonempty set, and $\mathcal{P}$ be its power set. Each element of $\mathcal{P}$ is called an event. Credibility measure is a set function from $\mathcal{P}$ to [0, 1]. In order to ensure that the set function has certain mathematical properties, Li and Liu (2006) provided the following four axioms:

Axiom 1. (Normality) $\mathbf{Cr}\{\Theta\} = 1$ for the universal set Θ.

Axiom 2. (Monotonicity) $\mathbf{Cr}\{A\} \le \mathbf{Cr}\{B\}$ for any events $A \subseteq B, A, B \in \mathcal{P}$.

Axiom 3. (Duality) $\mathbf{Cr}\{A\} + \mathbf{Cr}\{A^c\} = 1$ for any event $A \in \mathcal{P}$.

Axiom 4. (Maximality) $\mathbf{Cr}\{\cup_i A_i\} = \sup_i \mathbf{Cr}\{A_i\}$ for any collection of events $\{A_i\}$ with $\sup_i \mathbf{Cr}\{A_i\} < 0.5$.

If Cr is a credibility measure, the triplet $(\Theta, \mathcal{P}, \mathrm{Cr})$ is called a credibility space. A fuzzy variable ξ is a function from a credibility space $(\Theta, \mathcal{P}, \mathrm{Cr})$ to the set of real numbers $\Re$.

Definition 2.1 (Li, 2013)**.** Suppose that ξ is a fuzzy variable defined on the credibility space $(\Theta, \mathcal{P}, \mathrm{Cr})$. Then its credibility function is