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Continuous Optimization

Competitive facility location and design with reactions of competitors already in the market

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ABSTRACT

A new retail facility is to locate and its service quality is to determine where similar facilities of competitors offering the same goods are already present. The market share captured by each facility depends on its distance to customers and its quality, which is described by a probabilistic Huff-like model. In order to maximize the profit of the new facility, a two-stage method is developed, which takes into account the reactions of the competitors. In the quality decision stage, the competitive decision process occurring among facilities is modelled as a game, whose solution is given by its Nash equilibrium. The solution, which can be represented as functions of the location of the new facility, is obtained by analytical resolution of a system of equations in the case of one facility in the market or by polynomial approximation in the case of multiple facilities. In the location decision stage, an interval based global optimization method is used to determine the best location of the new facility. Numerical experiments on randomly generated instances demonstrate the effectiveness of the method.

1. Introduction

Location problems form a wide class of mathematical optimization problems. Our interest in this kind of problems is mainly due to various applications that arise not only in choosing facility locations, but also in problems concerning the location of hardware components for performing certain tasks [19]. One branch of location theory deals with the location of retails and other commercial facilities which operate in a competitive environment. The facilities compete for market share, with profit maximization as their objective.

It is generally agreed that the first paper on competitive facility location is Hotelling's paper [14] on duopoly in a linear market. Hakimi [10–12] formulated the problem in a network. Drezner [5] solved a single facility location on the plane. Their formulations are based on the assumption that each customer patronizes its closest facility. A more realistic approach was introduced by Huff [15,16], in which a more reasonable market share model was proposed. Many competitive location models are available in the literature; see for instance the survey papers of Eiselt and Laporte [7], Eiselt et al. [8] and Plastria and Carrizosa [22]. They vary in the ingredients which form a competitive location model. Recently, several papers studied the competitive facility location and design problem [9,29,24], generalizing the previous ones, which assume that the quality of each facility is given, by including the quality decisions in the problem. However, most of them don't take into account the reactions of the facilities already in the market.

In this paper we consider the problem of location and design of a new facility that will compete for customer demands with other facilities already in the market. When no competitor exists, the facility to be located will have the monopoly of the market in that area. However, if in the same area there already exist other facilities offering the same goods, the new facility will have to compete for the market. In this latter case, we have to take into account the reactions of the competitors.

For a Huff-like competitive facility location and design problem in the planar market, we investigate the case where a single new facility wants to enter in the market where similar facilities of the competitors are already present. We seek for the location and the service quality of the new facility that maximize its profit. The attraction of each facility is determined by a gravitational type model. Since the relocation of existing facilities is very expensive, we assume that the competitors can only improve their qualities to compete with the new facility in order to minimize their market share loss. The Huff-like competitive facility location and design problem was already treated by Fernández et al. [9] and Toth et al. [29], but they did not take into account the reactions of the competitors to the loss of their market shares.

The competitive facility location and design problem in this paper is formulated by using a two-stage decision model, similar to that of

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Tobin et al. [28] and Miller et al. [20]. This division into two stages is motivated by the fact that in the real world a facility chooses its location before its quality of service [28] and the competitive decision process among facilities occurs only in the quality decisions. In the quality decision stage, for any given location of the new facility, each facility seeks for its best quality as the best response to the other facilities' quality decisions. The competitive decision process occurring among facilities is modelled as a game. The solution of the game is given by its Nash equilibrium, which is represented as functions of the location of the new facility. In the location decision stage, the best location of the new facility is determined by maximizing its profit, based on the quality decisions of all facilities made in the quality decision stage. The best location is found by using an interval based global optimization algorithm.

In this paper, we examine the existence of the Nash equilibrium in the case of one facility already present in the market and the case of more than one facility in the market. In the first case, we can analytically obtain the best quality decisions (the Nash equilibrium) of both facilities as functions of the location of the new facility. In the second case, we cannot analytically obtain the best quality decisions of all facilities. Instead, we use a polynomial approximation method (regression method) to approximately obtain them.

Similar approaches can be found in the literature, for instance [28,20]. They considered a firm who seeks to establish manufacturing facilities on a network characterized by either competitive or oligopolistic economic competition. An economic equilibrium model was proposed to describe the existing competition on the network in terms of equilibrium prices, demands, production levels and shipments. The equilibrium of the model is specified by a variational inequality. To solve the problem, they proposed a bi-level approach with four steps. In the first step, the variational inequality is solved to obtain the equilibrium. In the second step, sensitivity analysis methods for variational inequalities are applied to the solution of the economic equilibrium model. The sensitivity analysis yields partial derivatives which represent how the equilibrium will change in response to changes in the market. In the third step, the partial derivatives are used to linearly approximate the reaction functions of the competitors. In the last step, after incorporating the reaction functions into the profit function of the locating facility, the resulting profit maximization problem is solved to determine the optimal location of the facility. The approach was demonstrated by an example model which assumes the market to be of spatial Cournot–Nash oligopoly.

Although this paper also adopts a two-stage approach, our work is quite different from the works of Tobin et al. [28] and Miller et al. [20]. Firstly, the problem we study is to locate a new retail facility on the plane and to determine its service quality, whereas their problem is to locate a new manufacturing plant on a network and to determine its production levels, shipment quantities and prices. Secondly, we have proposed general and effective methods for the quality and location decisions in our two-stage approach for the competitive location and design problem, but they did not propose general methods for the resolution of several decision problems arisen in their bi-level approach for the manufacturing facility location problem. Moreover, they proposed to use the partial derivatives generated by sensitivity analysis to linearly approximate the reaction functions of the competitors. Approximation method is effective only when the functions are linear or approximately linear. In our problem, the reaction functions have high nonlinearity and we need to use a more sophisticated approximation method such as the polynomial approximation approach proposed in this paper to well approximate the reaction functions.

The performance of our approach is evaluated through computational experiments on randomly generated instances. Computational results show that the approach can find a near-optimal solution which gives the location and quality of the new facility and the qualities of all other facilities for each instance tested. The computational time of the approach is reasonable.

The rest of the paper is organized as follows. The competitive facility location and design problem is described and formulated in Section 2. Section 3 introduces the principles of the two-stage method. The analysis of Nash equilibrium of the quality decision game and the polynomial approximation of the quality functions of all facilities are given in Sections 4 and 5, respectively. In Section 6, the best location decision of the new facility is found by using an interval branch and bound algorithm. In Section 7, computational results on the evaluation of the performance of the method are presented and analyzed. Section 8 concludes this paper with some remarks about the current work and perspectives for future research.

2. Problem formulation

A retail enterprise wants to locate a new facility in a planar area where customers and other similar retail facilities of its competitors are present. Customers are represented by demand points, with each demand point representing the customers located in a small area surrounding it. Each demand point has a buying power representing the combined purchase potential of all customers assigned to the demand point.

To estimate the market share of each facility, the model proposed by Huff [15,16] is adopted. In the model, the patronizing behaviour of customers is assumed to be probabilistic, that is each customer splits its buying power among the facilities in the market proportionally to their attractions to it. The attraction of each facility to a given demand point is proportional to the quality of the facility and inversely proportional to the distance between the demand point and the facility.

T. Drezner [4] and Z. Drezner [6] analyzed a location problem following Huff's market share model, in which the quality of the new facility is assumed to be given. Fernández et al. [9] generalized the previous work by deciding both the location and the quality of the facility to be located. Our work is a generalization of the work of Fernández by taking into account the reactions of competitors.

The following notation is used to describe the facility location and design model under study:

Indices

i	index of demand point, $i = 1, \dots n$
j	index of existing facility, $j = 1, \dots, m$

Variables

x	location of the new facility, $x = (x_1, x_2)$
α_0	quality level of the new facility, $\alpha_0 > 0$
α_j	new quality level of the jth existing facility, $\alpha_j > 0$, $\forall j = 1, \dots, m$

Data

α'_j	former quality level of the jth existing facility, $j = 1, \dots, m$
p_i	location of the ith demand point, $p_i = (p_i^1, p_i^2)$
f_j	location of the jth existing facility, $f_j = \left(f_j^1, f_j^2\right)$
w_i	buying power of the ith demand point
β_0	cost for increasing one unit of the quality level for the new facility
β_j	cost for increasing one unit of the quality level for the jth facility
γ_i	weight for the quality of a facility as perceived by the demand point i
λ_i	weight for the distance to a facility as perceived by the demand point i