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Decision Support

Decision making with interval probabilities

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ABSTRACT

Handling uncertainty by interval probabilities is recently receiving considerable attention by researchers. Interval probabilities are used when it is difficult to characterize the uncertainty by point-valued probabilities due to partially known information. Most of researches related to interval probabilities, such as combination, marginalization, condition, Bayesian inferences and decision, assume that interval probabilities are known. How to elicit interval probabilities from subjective judgment is a basic and important problem for the applications of interval probability theory and till now a computational challenge. In this work, the models for estimating and combining interval probabilities are proposed as linear and quadratic programming problems, which can be easily solved. The concepts including interval probabilities, interval entropy, interval expectation, interval variance, interval moment, and the decision criteria with interval probabilities are addressed. A numerical example of newsvendor problem is employed to illustrate our approach. The analysis results show that the proposed methods provide a novel and effective alternative for decision making when point-valued subjective probabilities are inapplicable due to partially known information.

1. Introduction

Subjective probabilities are used to reflect a decision-maker's belief, which are traditionally analyzed in terms of betting behavior with presumption that there is exactly one such price for the bet [5]. This presupposition could be problematic, if considering such situations that the individual is not allowed to say "I don't know enough". Moreover, the price that the individual is willing to take the bet may be different from the price that the individual may find attractive to offer such a bet. Clearly, it is better if the price stated has some range that reflects the judgment indifference of a person. In fact, Camerer and Weber suggest that a person may not be uncomfortable giving such precise bounds [2]. Moreover, Cano and Moral assert that very often, an expert is more comfortable giving interval-valued probabilities rather than point-valued probabilities, especially in the following cases [3]:

- When little information to evaluate probabilities is available.
- When available information is not specific enough.
- In robust Bayesian inference, to model uncertainty about a prior distribution.
- To model the conflict situation where several information sources are available.

There are many researches for imprecision probabilities [4,6,14,17,20,30,34,38]. Most of researches related to interval probabilities, such as combination, marginalization, condition, Bayesian inferences and decision assume that interval probabilities are known. Smithson [27] suggests lower and upper probability models based the anchoring-and-adjustment process for obtaining subjective probabilities that is initiated by Einhorn and Hogarth [8]. Imprecise Dirichelet (ID) model proposed by Walley gives posterior upper and lower probabilities satisfying invariance principle for making inference from multinomial data [35]. Dempster and Shafer define upper and lower probabilities, called plausibility and belief, respectively, based on a basic probability assignment [7,25]. Yager and Kreinovich suggest a formula to estimate the upper and lower bounds of interval probabilities from a statistical viewpoint [40]. However, eliciting interval-valued probabilities from subject still poses a computational challenge [1].

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This paper proposes a method for estimating interval probabilities for finite events. The method is based on the pairwise subjective comparisons of the likelihood of the events. It is well known that pairwise comparisons are comprehensively used in multiple criteria decision making (MCDM) problems with the common knowledge that eliciting indirect preference is less demanding of cognitive effort. For example, Greco et al. [10] propose a set of additive value functions based multiple criteria ranking models. The preference information provided by a decision maker is a set of pairwise comparisons on a subset of alternatives' set. Ordinal regression via linear programming (LP) is used for obtaining the set of all additive value functions compatible with the preference information. As an extension, a method called Generalized Regression with Intensities of Preference (GPIP) is proposed [9]. For characterizing the inherent inconsistency of a person's judgment on pairwise comparison, several approaches have been proposed for eliciting interval weights from a comparison matrix in the analytic hierarchy process (AHP) [23]. In the literature [24], a method for modeling the inconsistency of a person's judgment is brought forward. This method converts the entries of comparison matrices into extended regions, which are nonempty sets of weights bounded by linear constraints. By solving a series of linear programming problems, the upper and lower bounds of weights can be obtained. Sugihara et al. propose an interval regression based method (IRBM) for obtaining interval weights from the given interval comparison matrix [28]. This approach involves solving lower and upper approximation models. The lower approximation model captures the interval weight included by interval judgment, whereas the upper approximation model captures the interval weight including interval judgment. The crisp comparison matrix can be regarded as a special case of interval comparison matrix where only the upper approximation model is available. The detail about possibilistic regression models can be found in the literatures [15,29]. Wang and Elhag propose a goal programming (GP) method for interval weights from an interval comparison matrix [36]. This method is a variety of IRBM where instead of the inclusion relations in IRBM, a deviation vector is introduced to construct GP problem for obtaining the interval weights. In the literature [19], interval judgment is introduced into weighting procedures (SMART and SWING) for handling preference and information imprecision in MCDM. Based on the interval weights, the upper and lower bounds of the overall value of an alternative are obtained by LP problems.

In this research, the approaches for estimating interval probabilities are proposed with pairwise subjective comparisons of the likelihood of events. The linear programming and quadratic programming problems are employed to minimize the imprecision of judgment. The model for combining interval probabilities from different information sources is presented as a quadratic programming problem. Interval entropy, interval expectation, interval variance, and interval moment are studied in detail. The decision criteria with interval probabilities are given. The newsvendor problem for a new product is employed to illustrate our approach. Due to lack of market information on a new product, it is difficult to estimate the point-valued subjective probabilities of market demands. Using the proposed method, the interval probabilities of demands are obtained. The imprecision inherently existing in judging subjective probabilities is analyzed from the aspects of ignorance, interval entropy, interval expectation, and interval variance. The optimal order is obtained based on the partially ordered set of interval expected profits.

This paper is organized as follows: Section 2 provides some basic concepts and operations related to interval probabilities. In Section 3, the methods for estimating and combining interval probabilities are presented. In Section 4, as a numerical example, the newsvendor problem is considered. Finally, concluding remarks for this research are made in Section 5.

2. Interval probabilities

Let us consider a variable x taking its values in a finite set $X = \{x_1, \ldots, x_n\}$ and a set of intervals $L = \{W_i = [w_{*i}, w_i^*], i = 1, \ldots, n\}$ satisfying $w_{*i} \leqslant w_i^* \quad \forall i$. We can interpret these intervals as interval probabilities as follows.

Definition 1. The intervals $W_i = [w_{*i}, w_i^*], i = 1, \ldots, n$, are called the interval probabilities of X if for $\forall w_i \in [w_{*i}, w_i^*]$, there are $w_1 \in [w_{*1}, w_1^*], \ldots, w_{i-1} \in [w_{*i-1}, w_{i-1}^*], w_{i+1} \in [w_{*i+1}, w_{i+1}^*], \ldots, w_n \in [w_{*n}, w_n^*]$ such that

$$\sum_{i=1,\ldots,n} w_i = 1. \tag{1}$$

It can be seen from Definition 1 that the point-valued probability mass function is extended into the interval-valued function. Similar definitions called n-dimensional probability interval (n-PRI) (Definition 2.1, [37]) and feasible interval-valued probability distribution [13] have been given.

Lemma 1. *The set of intervals L satisfies (1) if and only if the following conditions hold:*

$$w_i^* + w_{*1} + \cdots + w_{*i-1} + w_{*i+1} + \cdots + w_{*n} \leqslant 1 \quad \forall i, \tag{2}$$

$$w_{*i} + w_1^* + \cdots + w_{i-1}^* + w_{i+1}^* + \cdots + w_n^* \geqslant 1 \quad \forall i. \tag{3}$$

Proof. See the appendix. □

It should be noted that Lemma 1 has been given as Theorem 2.2 by Weichselberger and Pohlmann [37]. It is clear that if there are only two interval probabilities $[w_{*1}, w_1^*]$ and $[w_{*2}, w_2^*]$ then $w_{*1} + w_2^* = 1$ and $w_1^* + w_{*2} = 1$ hold, and if we completely have no knowledge on X, we can express such kind of complete ignorance as $W_1 = W_2 = \cdots = W_n = [0, 1]$, which satisfies (2) and (3).

Let us consider a more general case where a set of intervals $L' = \{W_i' = [w_{li}, w_{ui}], i = 1, \ldots, n\}$ corresponding to a finite set $X = \{x_1, \ldots, x_n\}$ satisfies with the following inequalities

$$\sum_{i=1,\ldots,n} w_{li} \leqslant 1 \quad \forall i, \tag{4}$$

$$\sum_{i=1,\ldots,n} w_{ui} \geqslant 1 \quad \forall i. \tag{5}$$

It is clear that (4) and (5) are necessary conditions of (2) and (3), respectively. In other words, (2) and (3) are relaxed as (4) and (5), respectively. In this case, interval probabilities can be elicited from L' by the following linear programming problem