European Journal of Operational Research 251 (2016) 36–43

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Discrete Optimization

A two-stage solution method for the annual dairy transportation problem

Renaud Masson[a,b,*], Nadia Lahrichi[a,b], Louis-Martin Rousseau[a,b]

[a] CIRRELT, C.P. 6128, Succ. Centre-Ville, Montreal H3C 3J7, Canada
[b] Département de mathématiques et génie industriel, École Polytechnique de Montréal, Canada

ARTICLE INFO

Article history:
Received 15 October 2013
Accepted 26 October 2015
Available online 30 October 2015

Keywords:
Dairy transportation problem
Two-stage method

ABSTRACT

The annual dairy transportation problem involves designing the routes that collect milk from farms and deliver it to processing plants. The demands of these plants can change from one week to the next, but the collection is fixed by contract and must remain the same throughout the year. While the routes are currently designed using the historical average demand from the plants, we show that including the information about plants demands leads to significant savings. We propose a two-stage method based on an adaptive large neighborhood search (ALNS). The first phase solves the transportation problem and the second phase ensures that the optimization of plant assignment is performed. An additional analysis based on period clustering is conducted to speed up the resolution.

1. Introduction

In Canada, the dairy industry is ranked third in the agricultural sector (after grains & oilseeds and red meat). In 2011 the total net sales of dairy products generated $13.7 billion, representing 16.4% of the Canadian food and beverage sector. Most Canadian dairy farms are located in Ontario and Quebec. The dairy processing sector is concentrated: an 80 percent of the total raw milk produced in Canada is processed by three companies.

In Quebec, the *Producteurs de Lait du Québec* (PLQ), a coalition of dairy farmers, is responsible for planning the transportation of the milk produced in the province. The total annual cost of milk transportation from farms in Quebec to processing plants is more than $70 million. The problem considered in this paper is the design of the routes that collect milk from farms and deliver it to processing plants.

Currently the PLQ solves two distinct problems: an annual problem to design the routes that collect the milk and a weekly problem to assign the routes to the plants depending on the quantity of milk requested. Obviously the routes have a large impact on the weekly assignments. In this paper, we integrate the two problems by considering the varying demand throughout the year as we design the routes. To our knowledge, the only analogous contribution in the literature deals with the dairy transportation problem introduced by Lahrichi, Crainic, Gendreau, Rei, and Rousseau (2014). Although it handles the routing and the assignments to plants, it does not consider that the demand vary throughout the day therefore simplifying this aspect of the problem. In this paper, we aim to integrate this weekly variations to better plan the routes and the assignment of routes to plants. This problem will be referred to as the Annual dairy transportation problem in the remainder of the paper.

This paper makes the following two contributions. First, we introduce a new routing and assignment problem, the ADTP. Second we improve on the state-of-the-art method for the dairy transportation problem (DTP). Average savings with the new approach to solve the DTP are as considerable as 4 percent. We also show that using the weekly information during the annual design of routes leads to additional savings. The proposed method is very efficient for the ADTP and may be adapted to similar problems in other industries.

The remainder of this article is structured as follows. We first define the problem and present a review of related problems. We then describe the proposed two-stage adaptive large neighborhood search (ALNS) and finally discuss our numerical experiments and provide concluding remarks.

2. Problem statement

The problem introduced in this article differs in several ways from classical vehicle routing problems. The dairy transportation problem was introduced by Lahrichi et al. (2014). It involves the design of minimum-cost routes that collect milk from producers and deliver

* Corresponding author at: Département de mathématiques et génie industriel, École Polytechnique de Montréal, Canada. Tel.: +1 514 649 5357.

E-mail addresses: renaud.masson@cirrelt.ca, masson.renaud@gmail.com (R. Masson), nadia.lahrichi@cirrelt.ca (N. Lahrichi), louis-martin.rousseau@cirrelt.ca (L.-M. Rousseau).

http://dx.doi.org/10.1016/j.ejor.2015.10.058
0377-2217/

it to processing plants. The PLQ faces this problem every year when designing the contracts that determine which routes will be performed by each transporter.

We introduce the annual version of the dairy transportation problem (ADTP), which consists of including the variation of the demand. Indeed, during each week of the year, each plant specifies its demand for milk. The total quantity of milk that it receives at the end of the week must be close to the quantity requested (a tolerance is specified for each plant). This quantity is then split between each day (at least a predefined percentage of its weekly demand is assigned to ensure fairness between plants since most of them prefer to receive milk on weekdays rather than weekends.

The transportation problem is described as follows. We consider a heterogeneous fleet of capacitated vehicles located in various depots. Each producer is visited once by exactly one route that collects all its milk (split pickups are not allowed). The sequence of producers visited by a route has to be the same on every day of the planning horizon (fixed by contract), but the plants visited can change.

We present the mathematical model to formalize the problem the ADTP and the complicating constraints related to the weekly demand of the plants. Let F be the set of milk producers and P the set of processing plants. The set of weeks considered in the planning horizon is W. For each week $w \in W$, the demand of plant $p \in P$ is d_p^w. We denote by δ_p the tolerance for plant $p \in P$, and ξ_p% is the minimum percentage of its weekly demand that plant $p \in P$ must receive every day. Let K be the heterogeneous fleet of capacitated vehicles; each vehicle $k \in K$ has capacity q_k. Each producer $f \in F$ produces a quantity m_f of milk. Let D_w be the days of week $w \in W$; $D = \bigcup_{w \in W} D_w$ is the set of all the days of the planning horizon. O is the set of origin depots, and O' is the set of destination depots. The starting and ending depots of vehicle $k \in K$ are denoted $o(k)$ and $o'(k)$, and $N = F \cup O \cup O'$ is the set of all the nodes of the problem. The cost of the arc between nodes i and j is denoted $c_{i,j}$. We present this model to clearly state the problem that we are solving. To improve its readability, we do not linearize the nonlinear constraints (we do not solve this model directly).

The variables are as follows: $x_{i,j}^k$ is a binary variable indicating whether or not vehicle $k \in K$ goes from $i \in O \cup O' \cup F$ to $j \in O \cup O' \cup F$. The delivery cost of route $k \in K$ for day $d \in D$ is represented by continuous variable b_d^k. The quantity delivered by vehicle $k \in K$ to plant $p \in P$ on day $d \in D$ is represented by variable $l_{p,k}^d$. Finally, the binary variable $y_{p,d}^k$ indicates whether or not vehicle $k \in K$ visits plant $p \in P$ on day $d \in D$.

$$min \sum_{k \in K, i \in \{o(k)\} \cup F, j \in F} x_{i,j}^k c_{i,j} |D| + \sum_{k \in K, d \in D} b_d^k \quad (1)$$

The objective function (1) minimizes the distance covered every day to collect and deliver the milk.

$$\sum_{f \in F} x_{o(k),f}^k = 1 \qquad \forall k \in K \quad (2)$$

$$\sum_{i \in N} x_{i,o(k)}^k = 0 \qquad \forall k \in K \quad (3)$$

$$\sum_{i \in N} x_{o'(k),i}^k = 0 \qquad \forall k \in K \quad (4)$$

$$\sum_{f \in F} x_{f,o'(k)}^k = 1 \qquad \forall k \in K \quad (5)$$

Constraint (2) ensures that each vehicle leaves its starting depot only once. Constraint (3) ensures that no vehicle enters its starting depot, and constraint (4) ensures that no vehicle leaves its ending depot. Constraint (5) forces the vehicles to end their routes at their ending depots.

$$\sum_{k \in K, j \in F \cup \{o'(k)\}} x_{f,j}^k = 1 \qquad \forall f \in F \quad (6)$$

Constraint (6) ensures that each producer is visited exactly once.

$$\sum_{j \in F \cup \{o'(k)\}} x_{f,j}^k = \sum_{j \in F \cup \{o(k)\}} x_{j,f}^k \qquad \forall f \in F \quad (7)$$

Constraint (7) ensures vehicle flow conservation.

$$\sum_{f \in F, j \in F \cup \{o(k)\}} x_{f,j}^k m_f \le q_k \qquad \forall k \in K \quad (8)$$

Constraint (8) ensures that the vehicle capacities are respected.

$$y_{p,d}^k = 0 \Rightarrow l_{p,k}^d = 0 \qquad \forall p \in P, d \in D, k \in K \quad (9)$$

Constraint (9) prevents a plant from receiving milk from a vehicle on a given day, if the vehicle does not visit that plant.

$$\sum_{p \in P} l_{p,k}^d = \sum_{f \in F, j \in F \cup \{o'(k)\}} x_{f,j}^k m_f \qquad \forall d \in D, k \in K \quad (10)$$

Constraint (10) ensures that all the collected milk is delivered.

$$\sum_{k \in K} l_{p,k}^d \ge \xi_p \sum_{d' \in D_w, k \in K} l_{p,k}^{d'} \qquad \forall p \in P, w \in W, d \in D_w \quad (11)$$

$$\sum_{d \in D_w, k \in K} l_{p,k}^d \le (1 + \delta_p) d_p^w \qquad \forall p \in P, w \in W \quad (12)$$

$$\sum_{d \in D_w, k \in K} l_{p,k}^d \ge (1 - \delta_p) d_p^w \qquad \forall p \in P, w \in W \quad (13)$$

Constraint (11) ensures that each plant receives each day at least a given percentage of the weekly delivery. Constraints (12) and (13) ensure that the quantity received each week by each plant is sufficiently close to the quantity requested.

$$\sum_{i \in S, j \in S} x_{i,j}^k \le |S| - 1 \qquad \forall k \in K, S \subseteq F \cup O \cup O', |S| \ge 2 \quad (14)$$

Constraint (14) ensures subtour elimination.

$$b_d^k \ge x_{f,o'(k)}^k y_{p,d}^k (c_{f,p} + c_{p,o'(k)}) \qquad \forall k \in K, d \in D, f \in F \quad (15)$$

Constraint (15) determines the delivery cost of each route on each day.

$$x_{i,j}^k \in \{0, 1\} \qquad \forall k \in K, i \in \{o(k)\} \cup F, j \in F \cup \{o'(k)\} \quad (16)$$

$$y_{p,d}^k \in \{0, 1\} \qquad \forall k \in K, p \in P, d \in D \quad (17)$$

$$b_d^k \in \mathbb{R}^+ \qquad \forall k \in K, \forall d \in D \quad (18)$$

$$l_{p,k}^d \in \mathbb{R}^+ \qquad \forall k \in K, \forall d \in D, \forall p \in P \quad (19)$$

Constraints (16)–(19) are domain constraints.

We illustrate in Fig. 1 how the annual routes are constructed and how they are modified weekly. Fig. 1(a) shows a route starting at depot d_1, visiting farms f_1 through f_5 and delivering the milk collected in processing plant p_2. The daily cost of a route is the sum of (i) the distance covered by the vehicle during the collection (i.e, from d_1 to f_5), (ii) the distance between the last producer visited and the plant that is the farthest from the last producer (i.e, between f_5 and p_2), and (iii) the distance between that plant and the depot (i.e, between p_2 and d_1). The total cost of the solution is the sum of the daily route costs.

Fig. 1(b) illustrates the modification to the routes to better serve the demand of the plants. Due to the amount of milk requests by plants p_1 and p_2, the route has to be diverted to p_1. The sequence of producers stay identical in (a) and (b).