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Discrete Optimization

A formulation space search heuristic for packing unequal circles in a fixed size circular container

C. O. López [a], J. E. Beasley [b,*]

[a] Faculty of Sciences, National Autonomous University of Mexico, Mexico City, Mexico
[b] Mathematical Sciences, Brunel University, Uxbridge UB8 3PH, UK and JB Consultants, Morden, UK

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ABSTRACT

In this paper we consider the problem of packing unequal circles in a fixed size circular container, where the objective is to maximise the value of the circles packed. We consider two different objectives: maximise the number of circles packed; maximise the area of the circles packed.

For the particular case when the objective is to maximise the number of circles packed we prove that the optimal solution is of a particular form.

We present a heuristic for the problem based upon formulation space search. Computational results are given for a number of publicly available test problems involving the packing of up to 40 circles. We also present computational results, for test problems taken from the literature, relating to packing both equal and unequal circles.

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1. Introduction

The problem considered in this paper concerns the packing of unequal circles in a fixed size circular container. It can be easily understood by means of an example. Consider Fig. 1 which shows five circles (of radii 1,2,3,4,5) and a circular container (of radius 6). Suppose that we wish to maximise the number of smaller circles that we pack inside the larger circular container. Which of the smaller circles should we choose and where should they be positioned?

Here any packing chosen must satisfy the constraints that the packed circles lie fully inside the circular container and do not overlap. One possible solution is shown in Fig. 2. Here we have chosen to pack circles 1, 2 and 3; at the positions shown.

A related problem is to maximise the area of the smaller circles that we pack inside the larger circular container. One possible solution to this problem is shown in Fig. 3. Here we have chosen to pack circles 1 and 5; at the positions shown.

It is trivial to demonstrate that a solution which maximises the number of circles packed does not necessarily also maximise the area of the circles packed. For example, suppose we expand the example considered here to include a sixth circle of radius equal to the radius of the circular container. Clearly to maximise the area of the circles packed we would chose this sixth circle. But obviously a solution that involves just this single sixth circle is not optimal if we were interested in maximising the number of circles packed.

This paper is organised as follows. In Section 2 we discuss the literature relating to the circle packing problem with unequal circles, as well as the literature relating to formulation space search. We also discuss the practical applications of the problem considered in this paper, as well as what we believe the contribution of this paper to the literature to be. In Section 3 we present our formulation of the problem, whilst Section 4 proves that when maximising the number of circles packed the optimal solution has a particular form. Section 5 describes the algorithm proposed. In Section 6 we present computational results and finally in Section 7 we present conclusions.

2. Literature survey and contribution

In this section we review the work that has been done with respect to circle packing and unequal circles. It is important to note here that, for space reasons, we concentrate purely on literature relating to packing unequal circles. We also discuss the practical applications of the problem considered in this paper. We give a brief introduction to formulation space search (henceforth FSS) and the work that has been done using FSS. We also discuss what we believe to be the contribution of this paper to the literature.

2.1. Circle packing

A common feature of circle packing problems with unequal circles (i.e. circles with differing sizes) as considered in the literature is

* Corresponding author. Tel.: +44 1895266219.
E-mail addresses: claudia.lopez@ciencias.unam.mx (C.O. López), john.beasley@brunel.ac.uk, john.beasley@jbconsultants.biz (J.E. Beasley).

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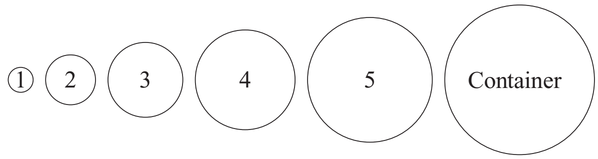

Fig. 1. Circles to be packed and the container.

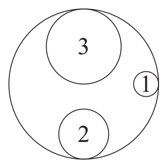

Fig. 2. Maximising the number of circles packed.

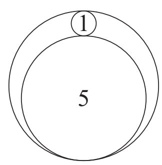

Fig. 3. Maximising the area of the circles packed.

that it is assumed that there exists a set of circles which all have to be packed into a container. ***In other words, there is no element of choice as to whether a circle is packed or not, it must be packed.*** This avoids introducing any combinatorial element into the problem which would complicate any solution approach.

In order to pack every unequal circle in a specified set typically the container is of variable dimension. So for example if the container is a circle it is of variable radius, leading to the natural optimisation problem: find the minimum circular container radius such that all circles can be packed. This problem has been extensively considered in the literature, e.g. see (Addis, Locatelli, & Schoen, 2008; Akeb & Li, 2006; Al-Mudahka, Hifi, & M'Hallah, 2011; Castillo, Kampas, & Pinter, 2008; Fu, Huang, & Lu, 2013; Grosso, Jamali, Locatelli, & Schoen, 2010; Hifi & M'Hallah, 2009, 2008a, 2008b; Huang, Fu, & Xu, 2013; Huang, Li, Li, & Xu, 2006; Huang, Li, & Xu, 2001; Huang & Zhan, 1982; Kallrath, 2009; Liu, Xue, Liu, & Xu, 2009a; Liu, Yao, Zheng, Geng, & Zhou, 2009b; López & Beasley, 2013; Lu & Huang, 2008; Wang, Huang, Zhang, & Xu, 2002; Zhang & Deng, 2005; Zhang & Huang, 2004a).

If the variable dimension container is a rectangle (of which the square is a special case) there are a number of differing objectives that have been adopted in the literature. So for the square a natural objective is to minimise the length of the side, this also minimises the perimeter and area of the square. For the rectangle natural objectives are to minimise the perimeter or minimise the area.

Alternatively for the rectangle we can regard one dimension as fixed and minimise the other dimension. Problems of this type are often referred to as strip packing problems (or as circular open dimension problems).

The problem of packing unequal circles into rectangular containers has also been considered in the literature, e.g. see (Akeb & Hifi, 2008, 2013; Castillo et al., 2008; George, George, & Lamer, 1995; He, Huang, & Yang, 2015; He & Wu, 2013; Huang, Li, Akeb, & Li, 2005; Kubach, Bortfeldt, & Gehring, 2009; López & Beasley, 2013; Stoyan & Yaskov, 2014; 2004; Zhang, Liu, & Chen, 2004b).

In this paper we introduce the element of choice as to whether a circle is packed or not. This introduction of a combinatorial element into the problem transforms the problem from a continuous nonlinear optimisation problem into a nonlinear optimisation problem involving zero-one (binary) variables. Such problems are more generally referred to as MINLPs (mixed-integer nonlinear programming problems).

To the best of our knowledge the problem considered in this paper, packing unequal circles in a fixed size circular container where there is an element of choice as to whether to pack a circle or not, has not been considered in the literature before. Indeed, as far as we aware, there is also no work in the literature relating to this problem with respect to any other containers with a circular element, such as rings and semicircles.

2.2. Practical applications

The circle packing problem has a long history and a wide variety of applications. An introduction to its history can be found in Szabó et al. (2007). Practical applications of the circle packing problem relating to: circular cutting, container loading, cylinder packing, facility dispersion and communication networks, facility and dashboard layout, are considered in Castillo et al. (2008).

The practical applications of the specific problem considered in this paper arise when, within the contexts discussed in Castillo et al. (2008), the circular container is of insufficient size to pack every circle. As such it becomes necessary to make a choice as to which circles to pack. The circular container may be of insufficient size due to economic considerations, or due to placement considerations (i.e. its size is limited as the container has to fit within a predefined space).

One practical example of the problem given in this paper of which we are aware relates to fibre optic cabling. Here the circular container is an outer protective layer within which we must pack a number of circular fibre optic cables. These cables are typically of a standard diameter, such as 250 microns and 900 microns. The question therefore is how many 250 micron cables, and how many 900 micron cables, can be packed within the circular container (and their positioning). This question can be answered using the heuristic given in this paper. For example, referring to Fig. 1, suppose we have a large number of circles of diameter 250 microns, and a large number of circles of diameter 900 microns. Then we have exactly the same problem as considered above in Fig. 1, which circles should be chosen and where should they be positioned in the circular container so as to maximise an appropriate objective.

2.3. Formulation space search

Formulation space search was first introduced by Mladenović, Plastria, and Urošević (2005) in the context of circle packing (formulated as a nonlinear optimisation problem). FSS was motivated by the remark in Mladenović et al. (2005) that, when solving a nonlinear problem with the aid of a solver, the solution may not be a local optimum, but a stationary point. Stationary points are those whose derivative is zero, but are neither minimum nor maximum. Different formulations of the same problem may have different characteristics that can be exploited to escape from stationary points, and possibly find better solutions. That is, whilst in one formulation we reach a stationary point, this may not be the case in another formulation. The main idea is to solve alternatively each formulation of the problem, once the solution is the same for all formulations then a good solution has been found. Applications of FSS relating specifically to circle packing can be found in López and Beasley (2011, 2013), Mladenović et al. (2005), Mladenović, Plastria, and Urošević (2007).

In López and Beasley (2011, 2013, 2014) FSS was expanded beyond the original concept of different stationary points in different formulations. That work exploits the fact that:

- because of the nature of the solution process in nonlinear optimisation we often fail to obtain a globally optimum solution from a single formulation; and so
- perturbing/changing the formulation and then resolving the nonlinear program may lead to an improved solution.

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