European Journal of Operational Research 251 (2016) 148–157

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Stochastics and Statistics

Optimal switching decisions under stochastic volatility with fast mean reversion[☆]

Andrianos E. Tsekrekos[a], Athanasios N. Yannacopoulos[b,*]

[a] Department of Accounting & Finance, Athens University of Economics & Business, Greece
[b] Department of Statistics, Athens University of Economics & Business, Greece

ARTICLE INFO

Article history:
Received 21 June 2014
Accepted 2 December 2015
Available online 12 December 2015

Keywords:
Optimal switching problems
Multi-scale stochastic volatility
Quasi-variational inequalities
Hysteresis

ABSTRACT

We study infinite-horizon, optimal switching problems for underlying processes that exhibiting "fast" mean-reverting stochastic volatility. We obtain closed-form analytic approximations of the solution for the resulting quasi-variational inequalities, that provide quantitative and qualitative results for the effects of multi-scale variability of the underlying process on the optimal switching rule. The proposed methodology is applicable to a number of operations research problems involving switching flexibility.

© 2015 Elsevier B.V. All rights reserved.

1. Introduction

An important class of problems arising in operations research are the so-called optimal switching problems, in which the objective is to find the optimal time to initiate or terminate a process, subject to uncertainty. This general formulation finds numerous applications in manufacturing and logistics (e.g. Benaroch, Webster, & Kazaz, 2012), networked systems, optimal control of energy systems (see for example Parpas and Webster, 2014; Ho and Parpas, 2014, and references therein), decision support (e.g. technology choice in Bobtcheff & Villeneuve, 2010), production management and capacity choice (see, e.g. Dixit, 1989; Trigeorgis, 1993; Pindyck, 1988; McDonald & Siegel, 1985), but also in fields such as natural resource management (see, e.g. Brennan & Schwartz, 1985; Paddock, Siegel, & Smith, 1988), transportation and shipping, etc. (Kavussanos, Tsekrekos, & Cullinane, 2011; Sødal, Koekebakker, & Aadland, 2008).

A key issue in all the above problems is the determination of the stochastic model that can represent the random evolution of the process in question, hereafter called the underlying. Depending on the particular concrete application in mind, the underlying can be interpreted in different ways, for instance a product price in manufacturing, an input price like the price of electricity in energy systems, a commodity price in natural resource management, etc.

A useful class of models for the evolution of the underlying is the family of Itô processes, which lead to convenient formulations of switching problems in terms of elliptic or parabolic quasi-variational inequalities (see for example Bensoussan & Lions, 1984; Brekke & Øksendal, 1994), the solution of which provides useful insights on the optimal switching decision rule. Most of these models adopt the assumption of constant volatility for the underlying process, mainly for analytical and numerical convenience. However, there is ample empirical evidence that often the underlying displays stochastic volatility effects, which may develop on different time scales (Eydeland & Wolyniec, 2003; Hikspoor & Jaimungal, 2008). In particular, one characteristic feature of volatility is that its mean-reversion rate is quite "fast", as compared to the time scale of evolution of the other state variables, a feature referred to as fast mean-reverting stochastic volatility. This has also been documented for the volatility of financial asset prices (Alizadeh, Brandt, & Diebold, 2002; Fouque, Papanicolaou, Sircar, & Sølna, 2003a, 2003b; Hillebrand, Fomby, & Terrell, 2006), and stochastic volatility has played a prominent role in the valuation and hedging of financial derivatives, explaining many stylized facts of such markets. A comprehensive review related to stochastic volatility in financial markets is Taylor (1994), and a multi-scale approach to the problem of hedging and pricing financial derivatives has been developed in Fouque, Papanicolaou, Sircar, and Sølna (2011), leading to increased recent activity in this field (see for

☆ We would like to thank Mark Shackleton, Stephen Taylor and Konstantinos Vasileiadis for providing comments on an earlier version of the paper. The paper has also benefited from the comments and suggestions of participants at the 19th annual international conference "Real Options: Theory meets Practice" that was held in Athens and Monemvasia. The first author gratefully acknowledges the financial support provided by the Basic Research Funding Program of the AUEB research centre (Project EP–1990–01). Usual disclaimers apply.

* Corresponding author. Tel.: +30 210 8203801.
E-mail addresses: tsekrekos@aueb.gr (A.E. Tsekrekos), ayannaco@aueb.gr (A.N. Yannacopoulos).

http://dx.doi.org/10.1016/j.ejor.2015.12.011
0377-2217/© 2015 Elsevier B.V. All rights reserved.

example Souza & Zubelli, 2011; Zhu & Chen, 2011a; 2011b; Chen & Zhu, 2012).

Despite the fact that asset and commodity prices have been documented to exhibit fast mean-reverting volatility, the study of its effects on optimal switching policies has been overlooked. It is the aim of this paper to examine optimal switching decisions under multi-scale stochastic volatility, and assess its quantitative and qualitative effects.

To accomplish this, we formulate and solve infinite-horizon, optimal switching problems driven by a general class of stochastic volatility models that exhibit fast mean-reversion. We employ the perturbation method of Fouque, Papanicolaou, and Sircar (2000) on the resulting quasi-variational inequalities, a fact that allows us to approximate the full problem with a sequence of simplified and manageable problems, each one offering a “correction” of different order to the decision rule corresponding to the constant-volatility model. These corrections, that are the effect of fast stochastic volatility, are derived in closed-form, allowing one to analytically approximate the solution of the general switching problem up to any desired order. Our analytic approach is important as it lends itself easily to comparative statics that are of value to decision-makers dealing with processes or projects that can be switched from and to an idle/active mode, contingent on state variables that exhibit random evolution on multiple scales. Moreover, as the full multi-scale optimal switching problem is difficult and tricky to handle by numerical methods, our analytic results offer useful benchmarks for the numerical analysis of the full problem. Our general results are illustrated in terms of the extension of the switching problem in Dixit (1989), for multi-scale volatility.

The rest of the paper is organized as follows: Section 2 starts by presenting a general optimal switching problem under fast mean-reverting stochastic volatility, and proceeds by providing a general perturbative framework for the analytic approximation of the solution to any desired order. In Section Section 3 we illustrate the method for the multi-scale volatility extension of the switching problem in Dixit (1989) and provide comments on the qualitative and quantitative effects of multi-scale volatility on the optimal policy. Finally, Section 4 concludes the paper.

2. Optimal switching policy under fast mean-reverting stochastic volatility

Consider a risk-neutral decision maker contemplating a project/process that can operate in two modes (active or idle) and which, depending on the mode, produces a stochastic flow payoff $R_q(P)$ ($q=0$ is idle, $q=1$ active), where P is a commodity or product price (underlying) that can be modeled by a diffusion process. Transition from one mode to the other can take place instantaneously and for an unlimited number of times, but at constant fixed costs, each time a transition decision is made.[1]

The problem we consider here, is that of finding the optimal switching decision rule, for a wide class of diffusion models for the price process, exhibiting dynamics on fast/slow time scales, and in particular providing semi-analytic approximations for the decision rule. This framework allows one to avoid the need for numerical treatment and provides important qualitative and quantitative information on the effect of different volatility dynamics on the optimal switching decision.

[1] The assumption of risk-neutrality is not crucial for the solution and it is made only for simplicity. The extension to a risk-averse process/project owner is straightforward and we make it available upon request from the authors. Equally non-crucial is the assumption of instantaneous transition from one mode of operation to the other. Switches between modes that take time to implement could be easily be accommodated, only at the cost of extra notation.

Our generic model for the underlying P of the project belongs to a general class of latent-factor stochastic volatility models and is given by the Itô process

$$dP_t = \mu P_t dt + f(Y_t) P_t dW_{1,t}, \quad P_0 = p \tag{1}$$

$$dY_t = \delta^{-2}(m - Y_t)dt + \frac{\nu\sqrt{2}}{\delta}\left(\rho dW_{1,t} + \sqrt{1-\rho^2}\,dW_{2,t}\right), \quad Y_0 = y, \tag{2}$$

where Y_t is a latent stochastic factor that drives the volatility through a general feedback term described by a smooth function $f : I \subset \mathbb{R} \to \mathbb{R}$, ($I$ compact). It is important to note that f need not be specified at this point. In the above, $(W_{1,t}, W_{2,t})'$ is a standard, two-dimensional Wiener process on a complete filtered probability space satisfying the usual conditions (Karatzas & Shreve, 1991), with $|\rho| < 1$ a constant correlation coefficient.

The small parameter δ plays an important role in our analysis and models the fact that the latent factor Y follows a mean-reverting process, whose dynamics are on a “faster” time scale than the dynamics of P. Note that the introduction of the latent factor Y and of the different time scales make the dynamics of P more realistic, (as suggested by the empirical evidence in Alizadeh et al., 2002; Fouque, Papanicolaou, Sircar, and Sølna, 2003b; Hillebrand et al., 2006; Hikspoor and Jaimungal, 2008, among others), but at the cost of not being able to obtain, unless f is very specific, a closed-form solution for P. Furthermore, the difference in the time scales creates problems in the numerical resolution of (1) and (2), which is of the form of a stiff stochastic differential equation.

The decisions to switch from one mode of operation to the other can be modeled by an adapted, finite variation, càglàd process Q taking values in $\{0, 1\}$, with $Q_t = 0$ or 1 if at time t, the process is idle or active, respectively. Let q denote the mode of operation at $t = 0$.

Let K_0, K_1 denote the fixed costs of *leaving* the idle or the active mode respectively. For any switching decision rule $Q = \{Q_t\}_{t\geq 0}$ and initial state (q, p, y), the present value of the project is

$$J_{(q,p,y)}(Q) = \mathbb{E}\Bigg[\int_0^\infty e^{-rs}[R_1(P_s)Q_s + R_0(P_s)(1-Q_s)]ds - \sum_{s\geq 0} e^{-rs}\Big[K_0(\Delta Q_s)^+ + K_1(\Delta Q_s)^-\Big]\Bigg], \tag{3}$$

where r is the (constant) risk-free rate of interest and by $(x)^\pm$ we denote the positive/negative part of x. The flow functions R_q are assumed to be concave and of the form $R_q(p) = a_q p + b_q + \mathfrak{R}_q(p)$ where functions $\mathfrak{R}_q$ satisfy the condition $\lim_{p\to 0}\frac{\mathfrak{R}_0(p)}{p} = \lim_{p\to+\infty}\frac{\mathfrak{R}_1(p)}{p} = 0$.

The objective is to maximize (3) over all possible switching rules Q and obtain the value function,

$$V_q^\delta(p, y) := \sup_Q J_{(q,p,y)}(Q), \tag{4}$$

where the superscript δ is used to emphasize the dependence of the value function on the small parameter δ.

The above general model provides a convenient and flexible framework for the treatment of a wide class of switching problems encountered in the operations research and management science literature. Just to provide a few indicative examples, in the context of supply-chain contracts, several papers (e.g. Wagner & Friedl, 2007; Löffler, Pfeiffer, & Schneider, 2012) analyze supplier-switching options, that allow firms to switch suppliers and/or flexibly adjust their order quantity over time in the face of uncertainty in the underlying state process (e.g. the exchange rate in Kamrad & Siddique, 2004). In manufacturing (Kazaz, Dada, & Moskowitz, 2005; Kogut & Kulatilaka, 1994), as well as in service processes (Ellram, Tate, & Billington, 2008), optimal switching between in-sourcing and out-sourcing for

Download English Version:

https://daneshyari.com/en/article/480637

Download Persian Version:

https://daneshyari.com/article/480637

Daneshyari.com