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Interfaces with Other Disciplines

Portfolio optimization under loss aversion

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ABSTRACT

We present an integrated methodological approach for selecting portfolios. The proposed methodology is focused on incorporation of investor's preferences in the Mean-Risk framework. We propose a risk measure calculated with the downside part of the portfolio return distribution which, we argue, captures better the practical behavior of the loss-averse investor. We establish its properties, study the link with stochastic dominance criteria, point out the relations with Conditional Value at Risk and Lower Partial Moment of first order, and give the explicit formula for the case of scenario-based portfolio optimization. The proposed methodology involves two stages: firstly, the investment opportunity set (efficient frontier) is determined, and secondly, one single preferred efficient portfolio is selected, namely the one having the highest Expected Utility value. Three classes of utility functions with loss aversion corresponding to three types of investors are considered. The empirical study is targeted on assessing the differences between the efficient frontier of the proposed model and the classical Mean-Variance, Mean-CVaR and Mean-LPM1 frontiers. We firstly analyze the loss of welfare incurred by using another model instead of the proposed one and measure the corresponding gain/loss of utility. Secondly, we assess how much the portfolios really differ in terms of their compositions using a dissimilarity index based on the 1-norm. We describe and interpret the optimal solutions obtained and emphasize the role and influence of loss aversion parameters values and of constraints. Three types of constraints are studied: no short selling allowed, a certain degree of diversification imposed, and short selling allowed.

1. Introduction

Beginning with the seminal work of Markowitz (1952), the *Mean-Variance* model has been the main paradigm for the portfolio selection. To account for the obvious asymmetric nature of risk, there are a number of proposals, such as Roy's (1952) safety first technique that was decisive for the wide recognition of the fact that only downside risk is relevant for the investor. Markovitz (1959) proposes two semivariance measures computed only with the returns below the expected return, respectively below a given target return, also discussed in Markowitz (2012). The research on downside risk measures continued with the development of the Lower Partial Moment (LPM) introduced by Bawa (1975) and Fishburn (1977) and further developed and analyzed by Bawa and Lindenberg (1977), Harlow and Rao (1989), Grootveld and Hallerbach (1999), just to name a few. But nowadays, the financial industry uses extensively quantile-based risk measures as risk management tools, especially the Value at Risk (VaR), see for example Jorion (1997) and Dowd (1998). The ease and intuitiveness of VaR are counterbalanced by its mathematical properties and difficulties in numerical optimization. The Conditional Value at Risk was introduced by Rockafellar and Uryasev (2000) and has been defined for general distributions in Rockafellar and Uryasev (2002). It is a coherent measure of risk in the sense of Artzner, Delbaen, Eber, and Heath (1999), see Pflug (2000), Rockafellar and Uryasev (2002) and also Acerbi and Tasche (2002) and Stoyanov, Racheva-Iotova, Rachev, and Fabozzi (2010). Given its qualities, CVaR is a source of further developments, see for example Pavlikov and Uryasev (2014).

The alternative approach traditionally used in modeling the portfolio selection problem is the Expected Utility Theory (EUT) developed by Von Neumann, Morgenstern, and Princeton (1947). EUT is based on the axiomatic model of risk-averse preferences. Empirical evidence suggesting that inconsistencies may occur in EUT framework can be found in the papers of Camerer (2001), Starmer (2000), Hershey and Schoemaker (1985) and Rabin (2000). A distinct class of models of choice between random variables closely connected to EUT is the Stochastic Dominance (SD) rules, see Levy's survey (1992). As opposed to the Mean-Risk framework where each random variable is described by two scalars, SD provides a different approach: random variables are compared by pointwise comparison of functions constructed from their distribution functions. The tractability of models based on SD has been of a permanent concern, see

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the advances achieved in Dentcheva and Ruszczynski (2006), Roman, Darby-Dowman, and Mitra (2006), Fábián, Mitra, and Roman (2011), just to name a few.

In their papers from 1979 and 1992, Kahneman and Tversky present evidence that the decisions of actual investors depart not only from the assumed rationality of EUT, but also from the Mean-Variance model in many ways. They show that investors are loss averse: they consider the deviations of their terminal wealth as gains and losses starting from a reference level θ , and they react differently to gains than to losses. There is a variety of reasons triggered by real-life constraints explaining how one ends up having a critical level of potential losses, see for example Borgonovo and Gatti (2013) who investigated the consequences of including covenant breach in the risk analysis of large industrial projects, and provided an objective view on their effect and significance. There are many papers dealing with loss aversion: Benartzi and Thaler (1995) and Barberis, Huang, and Santos (2001) use numerical techniques to solve the portfolio problem of loss averse investors, Gomes (2005) provides an exact solution in a model with two states of the world, Berkelaar, Kouwenberg, and Post (2004) extend these results toward closed form solutions assuming a complete markets setting, Lien (2001) studies the implications of loss aversion for futures hedging, Zakamouline and Koekebakker (2009) show that a decision maker exhibits more than loss aversion, Fortin and Hlouskova (2011) and Best, Grauer, Hlouskova, and Zhang (2014) study the asset allocation for the particular case of linear loss-averse investor.

Our goal is to propose an alternative methodology for defining, measuring and optimizing risk that addresses some of the conceptual shortcomings of the *Mean-Risk*framework such as the disregard of investor's attitude toward risk and implicit assumption of neutrality to loss aversion. The key in our proposed methodology is a risk measure called *ESLA* that, for continuous return distribution functions, can be represented in terms of the conditional expectation of the distribution tail, where the tail is determined by the critical return level θ characterizing the loss-averse investor. The idea of isolating several damage ranges on which is based the partitioned multiobjective risk method introduced by Asbeck and Haimes (1984) was applied to real-life problems such as groundwater contamination, dam safety, flood warning and evacuation, navigation systems, software development, project management, risk management in agriculture, see Haimes and Santos (2009) and the references therein, and also to portfolio selection under the assumption of normal distributions in Santos and Haimes (2004).

The contributions of this paper can be summarized as follows.

In Section 2 we define *ESLA*, show the relations of the proposed risk measure with CVaR, and LPM of first order (LPM_1), establish its properties, study the link with stochastic dominance criteria, and discuss practical aspects regarding the calculation in the case of scenario-based portfolio optimization.

In Section 3 the portfolio selection methodology in two stages in which investor's loss aversion is fully taken into consideration is described: firstly, the investment opportunity set is determined (in our case, the *Mean-ESLA* efficient frontier), and secondly, one single preferred portfolio out of the entire frontier is selected – the procedure applied for this final selection was also used in Kroll, Levy, and Markowitz (1984), Levy and Levy (2004), De Giorgi and Hens (2009), Hens and Mayer (2014). Three types of investors characterized by different classes of utility functions with loss aversion are considered.

In Section 4 the empirical study is targeted on assessing the differences between the *Mean-ESLA* efficient frontier and the classical *Mean-Variance*, *Mean-CVaR*$_\alpha$ and *Mean-LPM*$_1$ frontiers. Firstly, we measure the loss of welfare incurred by using another model instead of the proposed one using a proximity index similar to that defined by Kroll et al. (1984) and Hens and Mayer (2014) in the *EU* framework. Secondly, to assess how much the portfolios really differ in terms of their compositions, we use a dissimilarity index based on the l_1 norm. We emphasize the role and influence of loss aversion parameters values and constraints.

In Section 5 we present the conclusions of our study.

2. *Mean-Risk* portfolio optimization with loss aversion

2.1. Definition and motivation of Expected Shortfall with Loss Aversion

Several definitions of loss aversion have been put forward in the literature, see for example Abdellaoui, Bleichrodt, and Paraschiv (2007), but here we consider the most usual one characterized by two parameters:

(i) the critical return level θ where the perception changes because of the passage from the outcomes perceived as gains to those perceived as losses,
(ii) the coefficient of loss aversion λ used to capture the fact that losses are more painful than equivalent gains even when the threshold θ is only slightly exceeded.

In order to measure in the most adequate way the risk in the presence of loss aversion, we should pay attention to two important aspects.

(1) **The critical threshold** (representing the upper bound of the tail of the return distribution used in the definition of the downside risk measure). What choice is more appropriate for a loss-averse investor? A threshold θ (loss aversion parameter) defined ex ante that remains constant during the optimization step? Or a variable one $\vartheta(R(\boldsymbol{x}))$, changing with each portfolio return distribution $R(\boldsymbol{x})$ such as $\mathrm{VaR}_\alpha(R(\boldsymbol{x}))$ for $\mathrm{CVaR}_\alpha(R(\boldsymbol{x}))$? Since we are concerned with the loss-averse investor, we argue that the natural choice is the constant parameter θ for all portfolios. Choosing to work with a variable threshold $\vartheta(R(\boldsymbol{x}))$ leads to ignoring the fact that the loss-averse investor has a clear image of his/her financial situation and knows for sure where the cut should be done. Moreover, changing the threshold $\vartheta(R(\boldsymbol{x}))$ at each portfolio means acting inconsistently (from the loss-averse investor's viewpoint) during the same cycle of optimization in the following sense:

- For portfolios for which $\vartheta(R(\boldsymbol{x})) > \theta$, returns between θ and $\vartheta(R(\boldsymbol{x}))$ are taken into account when calculating the risk measure despite the fact that the investor perceives them as manageable losses. For these portfolios, it is considered that the investor has an untrue *excessive degree of loss aversion*.
- For portfolios for which $\vartheta(R(\boldsymbol{x})) < \theta$, risk is not properly managed: returns ranging from $\vartheta(R(\boldsymbol{x}))$ to θ (subjectively perceived as losses by the particular investor) are simply ignored. For these portfolios the optimization step is performed as if the investor is *neutral from the loss-aversion viewpoint.*

(2) **The distribution used**. We argue that the conditional distribution of returns below the critical level are far more eloquent than the portfolio return distribution. Indeed, for the loss-averse investor for which the loss values breaching the critical threshold even with small probabilities are very important, the conditional distribution gives the proper weights to these high losses, while, when using the return distribution their magnitude is hidden to some extent.

General framework. Let n be the number of stocks used to build portfolios. The key random inputs in the portfolio problem are the stocks returns denoted by $r(\omega) = (r_1(\omega), ..., r_n(\omega))'$, $\omega \in \Omega$ or simply by r (we use a bold symbol for vectors). The set Ω represents the set of future states of knowledge and has the mathematical structure of a probability space with a probability measure P for comparing the likelihood of future states ω. Let $R(\boldsymbol{x}) = \boldsymbol{x}'\boldsymbol{r}$ be the return of the portfolio $\boldsymbol{x} \in X \subset \mathbb{R}^n$, where X is the set of available portfolios defined by the budget constraint and positivity constraints meaning that no short sell are allowed

$$X = \left\{\boldsymbol{x} = (x_1, ..., x_n)' \in \mathbb{R}^n | \boldsymbol{x}'\mathbf{1} = 1,\ x_i \geq 0,\ i = 1, ..., n\right\}, \quad (1)$$