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Interfaces with other disciplines

Productivity measurement in radial DEA models with a single constant input

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ABSTRACT

We consider productivity measurement based on radial DEA models with a single constant input. We show that in this case the Malmquist and the Hicks–Moorsteen productivity indices coincide and are multiplicatively complete, the choice of orientation of the Malmquist index for the measurement of productivity change does not matter, and there is a unique decomposition of productivity change containing two independent sources, namely technical efficiency change and technical change. Technical change decomposes in an infinite number of ways into a radial magnitude effect and an output bias effect. We also show that the aggregate productivity index is given by the geometric mean between any two periods of the simple arithmetic averages of the individual contemporaneous and mixed period distance functions.

1. Introduction

Radial DEA models with a single constant input have gained increasing popularity in recent years for use in situations in which the relative performance of units is evaluated with reference to the outputs they produce or the services they provide and without reference to the resources they consume in the process. Applications of the single constant input model generally fit into four areas, each involving static performance evaluation.

One area that covers a wide range of applications, noted by Yang, Shen, Zhang, and Liu (2014), occurs when ratio variables such as GDP per capita, output per hectare, value added per employee or a firm's revenue/cost ratio are used to evaluate performance, and the underlying data do not allow splitting ratio variables into numerators (outputs) and denominators (inputs). In this case the (desirable) ratio variables become outputs and there is a single constant input.

A second area is performance evaluation relative to best practice or to targets set by management. An early example was provided by Lovell and Pastor (1997), who analyzed target setting for bank branches. More recent examples include Halkos and Salamouris (2004) for evaluating the financial performance of Greek commercial banks; Wang, Lu, and Lin (2012) on bank holding company performance; Odeck (2005, 2006) for road traffic safety units; Soares de Mello, Angulo-Meza, and Branco da Silva (2009) for ranking the performance of countries in the Olympic Games; Lo (2010) for Kyoto Protocol target achievement; Liu, Zhang, Meng, Li, and Xu (2011) for the performance of Chinese research institutes; and Bezerra Neto, Christina, Porto, Gomes, and Cecílio Filho (2012) for agro-economic indices in polyculture.

A rapidly growing third area is the construction of composite indicators. Early examples include Thompson, Singleton, Thrall, and Smith (1986) and Takamura and Tone (2003) for comparative site evaluation. More recent examples include Cherchye, Moesen, and Van Puyenbroeck (2004) for macroeconomic indicators of country performance; Mizobuchi (2014) for the OECD Better Life Index; Guardiola and Picazo-Tadeo (2014) for life satisfaction indices; Zafra-Gomez and Muñiz Pérez (2010) and Lin, Lee, and Ho (2011) for local government performance evaluation; Murias, deMiguel, and Rodriguez (2008) for an educational quality indicator; Despotis (2005) for revising the Human Development Index; Lauer, Lovell, Murray, and Evans (2004) on the performance of the world health system; and Bellenger and Herlihy (2009), Lo (2010), Rogge (2012), Sahoo, Luptacik, and Mahlberg (2011), Zhou, Ang, and Poh (2007) and Zanella, Camanho, and Dias (2013) for environmental and ecological performance indicators.

A fourth area employs DEA as a multiple criteria decision analysis (MCDM) tool. Examples include Hadi-Vencheh (2010), Ramanathan (2006), Zhou and Fan (2007) and Chen (2011) for inventory classification, Seydel (2006) and Sevkli, Koh, Zaim, Dermibag, and Tatoglu (2007) for supplier selection, Lee and Kim (2014) and Charles and Kumar (2014) for service quality evaluation, and Yang et al. (2014) for

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the performance of Chinese cities and the performance of research institutes in the Chinese Academy of Sciences.

In this paper we take the use of the radial DEA models with a single constant input one step further by considering their potential use in inter-temporal performance evaluation by means of a pair of technology-based productivity indices, the Malmquist and the Hicks–Moorsteen indices.[2] In particular, (a) we compare the Malmquist and the Hicks–Moorsteen productivity indices for the single constant input model; (b) we develop a new decomposition of the sources of productivity change in this case and (c) we explore the aggregation of productivity changes from individual to group level. We show that in the single constant input case the Malmquist and the Hicks–Moorsteen productivity indices coincide, without having to impose restrictions on the structure of technology, and the orientation of the Malmquist productivity index does not matter. We also show that there is a unique decomposition of productivity change containing two independent components, technical efficiency change and technical change. Technical change decomposes in an infinite number of ways into a radial magnitude effect and an output bias effect. In addition, we show that the aggregate (group) productivity index equals the geometric mean between any two periods of the simple (un-weighted) arithmetic averages of the individual contemporaneous and mixed period distance functions.

In relating the above results with those previously presented in the literature note the following: *first*, the Malmquist and the Hicks–Moorsteen productivity indices coincide not only when restrictions are imposed on the structure of technology, such as a single input or a single output and constant returns to scale, as was claimed by Bjurek (1996), or constant returns to scale and inverse homotheticity, as was shown by Färe, Grosskopf, and Roos (1996)[3], or constant returns to scale and technological stagnation, as was shown by O'Donnell (2012, p. 258), or constant returns to scale and Hicks-neutral technical change, as was shown by Mizobuchi (2015), but also in the case of a single constant input. *Second*, it is not only the case of a global constant-returns-to-scale technology that there is a unique decomposition of productivity change but also the case of a single constant input. *Third*, it seems that the single constant input model is the only known case that the geometric mean between any two periods of the simple arithmetic averages of the individual contemporaneous and mixed period distance functions provides a consistent measure of aggregate productivity change by means of the Malmquist productivity index.

Although we explicitly consider an output-oriented model with a single constant input, the results can easily be extended to an input-oriented model with a single constant output. And although we motivated the exercise with empirical examples in which a single constant input is plausible, an extension to the case of a single constant output is also easily motivated.[4]

2. The main results

The Malmquist and the Hicks–Moorsteen indices are the two technology-based productivity indices that complement the set of price-based productivity indices (e.g., Fisher and Törnqvist), and their main advantage is that their measurement does not require price data. The former is expressed in terms of distance functions defined on the benchmark technology and the latter in terms of distance functions defined on the best practice technology. The two characterizations of technology differ in their returns to scale properties, and also in their technical change properties.

The output- and input-oriented Malmquist productivity indices are defined as:

$$M_O = \left[\frac{\tilde{D}_O^t\left(x^{t+1}, y^{t+1}\right)}{\tilde{D}_O^t\left(x^t, y^t\right)} \frac{\tilde{D}_O^{t+1}\left(x^{t+1}, y^{t+1}\right)}{\tilde{D}_O^{t+1}\left(x^t, y^t\right)} \right]^{1/2} \quad (1a)$$

$$M_I = \left[\frac{\tilde{D}_I^t\left(y^t, x^t\right)}{\tilde{D}_I^t\left(y^{t+1}, x^{t+1}\right)} \frac{\tilde{D}_I^{t+1}\left(y^t, x^t\right)}{\tilde{D}_I^{t+1}\left(y^{t+1}, x^{t+1}\right)} \right]^{1/2}, \quad (1b)$$

in which $x \in R_+^N$ and $y \in R_+^M$ are respectively input and output quantity vectors, and $\tilde{D}_O(x,y) = \min\{\lambda: (x,y/\lambda) \in \tilde{T}\}$ and $\tilde{D}_I(y,x) = \max\{\delta: (y,x/\delta) \in \tilde{T}\}$ are respectively output and input distance functions defined on a *benchmark* technology $\tilde{T} = \{(y,x): x \text{ can produce } y\}$ that exhibits (global) constant returns to scale. These distance functions are defined on data and technology from the same time period, in which case $\tilde{D}_o^t(x^t,y^t) \leq 1$ and $\tilde{D}_i^t(y^t,x^t) \geq 1$, and also data and technology from adjacent time periods, in which case $\tilde{D}_o^t(x^{t+1},y^{t+1}) \gtreqless 1$ and $\tilde{D}_i^t(y^{t+1},x^{t+1}) \gtreqless 1$ (i.e., data from one period may not be feasible with technology from the other period). The Malmquist productivity indices were introduced and named by Caves, Christensen, and Diewert (1982) but with distance functions defined on a *best practice* technology allowing for variable returns to scale. Grifell-Tatjé and Lovell (1995) showed however that this formulation prevents economies of size and diversification from contributing to productivity change, and it is now standard practice to define Malmquist productivity indices as in (1a) and (1b), because this formulation allows economies of size and diversification to contribute to productivity change. It does so by distinguishing the benchmark technology satisfying constant returns to scale from the best practice technology allowing for variable returns to scale, with deviations between the two reflecting the presence of economies of size and diversification and nothing else.

The non-oriented Hicks–Moorsteen productivity index is defined as the ratio of Malmquist output and input quantity indices, viz.:

$$HM = \frac{Q_y}{Q_x} = \frac{\left[\frac{D_O^t\left(x^t, y^{t+1}\right)}{D_O^t\left(x^t, y^t\right)} \frac{D_O^{t+1}\left(x^{t+1}, y^{t+1}\right)}{D_O^{t+1}\left(x^{t+1}, y^t\right)} \right]^{1/2}}{\left[\frac{D_I^t\left(x^{t+1}, y^t\right)}{D_I^t\left(x^t, y^t\right)} \frac{D_I^{t+1}\left(x^{t+1}, y^{t+1}\right)}{D_I^{t+1}\left(x^t, y^{t+1}\right)} \right]^{1/2}}, \quad (2)$$

in which $D_O(x,y)$ and $D_I(y,x)$ are respectively output and input distance functions defined as above but on the *best practice* technology $T = \{(y,x): x \text{ can produce } y\}$ that allows for variable returns to scale. These distance functions also are defined on data and technology from the same time period, and data and technology from adjacent time periods. This index was introduced by Bjurek (1994, 1996), who did not however give it its popular name. Because HM is expressed as the ratio of an output quantity index to an input quantity index, Bjurek (1994, 1996) called it, prophetically, "The Malmquist Total Factor Productivity Index". O'Donnell (2012, p. 257) characterizes HM as a "multiplicatively complete" productivity index because it is expressed as the ratio of an output quantity index to an input quantity index, with both indices being non-negative, non-decreasing and linearly homogeneous, and notes that the Malmquist productivity indices M_O and M_I do not share this desirable property, which implies that they cannot always be interpreted as measures of productivity change.

2.1. Measurement

We assume now that $x \in R_+^1$, and that the single input is constant, both across producers, the context Lovell and Pastor (1999) exam-

[2] Both Odeck (2005, 2006) and Lin, Lee and Ho (2011) used the single constant input model in an intertemporal context. However neither study provides any reasoning behind the standard decomposition of the Malmquist productivity index that involves no scale-related effect. As we show below, however, this is an inherent part of the radial single constant input model.

[3] Førsund (1997) has shown that these restrictions on the structure of technology coincide with distance functions introduced in Section 2 below satisfying homogeneity, identity, separability, proportionality and monotonicity properties.

[4] In the original version of this paper we extended the single constant input framework to a multiple constant input framework. However two reviewers have persuaded us that the extension is economically difficult to motivate and is mathematically trivial.