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Interfaces with Other Disciplines

Modeling and forecasting exchange rate volatility in time-frequency domain[☆]

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ABSTRACT

This paper proposes an enhanced approach to modeling and forecasting volatility using high frequency data. Using a forecasting model based on Realized GARCH with multiple time-frequency decomposed realized volatility measures, we study the influence of different timescales on volatility forecasts. The decomposition of volatility into several timescales approximates the behaviour of traders at corresponding investment horizons. The proposed methodology is moreover able to account for impact of jumps due to a recently proposed jump wavelet two scale realized volatility estimator. We propose a realized Jump-GARCH models estimated in two versions using maximum likelihood as well as observation-driven estimation framework of generalized autoregressive score. We compare forecasts using several popular realized volatility measures on foreign exchange rate futures data covering the recent financial crisis. Our results indicate that disentangling jump variation from the integrated variation is important for forecasting performance. An interesting insight into the volatility process is also provided by its multiscale decomposition. We find that most of the information for future volatility comes from high frequency part of the spectra representing very short investment horizons. Our newly proposed models outperform statistically the popular as well conventional models in both one-day and multi-period-ahead forecasting.

1. Introduction

In contrast to the conventional framework of a generalized autoregressive conditional heteroscedasticity (GARCH) model, volatility is directly observed and can be used for forecasting when high frequency data are applied.[1] While Hansen and Lunde (2005) argue that GARCH(1,1) can hardly be beaten by any other model, recent active research shows that with help of high frequency measures, we can improve the volatility forecasts significantly. Mcmillan and Speight (2012) for example utilize intra-day data and show that we can obtain forecasts superior to forecasts from GARCH(1,1). Louzis, Xanthopoulos-Sisinis, and Refenes (2013) assess the informational content of alternative realized volatility estimators using Realized GARCH in Value-at-Risk prediction. We extend this line of research by investigating the importance of disentangling jump variation and integrated variance in recently developed framework, which combines appeal of a widely used GARCH(1,1) and high frequency data. Moreover, we employ recently developed multiscale estimators which decompose volatility into several investment horizons[2] and allow us to study the influence of intraday investment horizons on the volatility forecasts.

Traders on financial markets make their decisions over different time horizons, for example, minutes, hours, days, or even longer such as months and years (Corsi, 2009; Gençay, Selcuk, & Whitcher, 2005; LeBaron, 2001; Ramsey, 2002). Nevertheless, majority of the empirical literature studies the relationships in the time domain only aggregating the behavior across all investment horizons. A notable exception is the Heterogenous Autoregressive approach (HAR) for realized volatility proposed by Corsi (2009). Although staying in the

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[1] A vast quantity of literature on several aspects of estimating volatility using high frequency data is nicely surveyed by McAleer and Medeiros (2008).

[2] An investment horizon refers to the length of time that an investor expects to hold a security or portfolio.

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time domain solely, Corsi (2009) builds his model on the idea of the investors' heterogeneity.

In our work, we ask if wavelet decomposition can provide better insight into the foreign exchange volatility modeling and forecasting.[3] Wavelets are often successfully used as a de-noising tool (Haven, Liu, & Shen, 2012; Sun & Meinl, 2012). One particularly appealing feature of wavelets is that they can be embedded into stochastic processes, as shown by Antoniou and Gustafson (1999). Thus we can conveniently use them to extend the theory of realized measures to obtain decomposed volatility as shown by Fan and Wang (2007), or Barunik and Vacha (2015). One of the common issues with the interpretation of wavelets in economic applications is that they are filter, thus they can hardly be used for forecasting in econometrics. Models based on wavelets are often outperformed by simple benchmark models, as shown by Fernandez (2008). Rather, they can provide a useful "lens" into the spectral properties of the time series. Our wavelet-based estimator of realized volatility uses wavelets only to decompose the daily variation of the returns using intraday information, hence the problem with forecasts is no longer an issue. As wavelets are used to measure realized volatility at different investment horizons, we can construct a forecasting model based on the wavelet decomposed volatility conveniently.

Several attempts to use wavelets in the estimation of realized variation have emerged in the past few years. Høg and Lunde (2003) were the first to suggest a wavelet estimator of realized variance. Capobianco (2004), for example, proposes to use a wavelet transform as a comparable estimator of quadratic variation. Subbotin (2008) uses wavelets to decompose volatility into a multi-horizon scale. One exception is the work of Fan and Wang (2007), who were the first to use the wavelet-based realized variance estimator together with the methodology for estimation of jumps. Barunik and Vacha (2015), revisit and extend this work and using large Monte Carlo study show that their estimator improves forecasting of the volatility substantially when compared to other estimators. Moreover, Barunik and Vacha (2015) attempt to use the estimators to decompose stock market volatility into several investment horizons in a non-parametric way.

Motivated by previous results, this paper focuses on proposing a model which will improve the modeling and forecasting of foreign exchange volatility. Similarly to Lanne (2007); Andersen, Bollerslev, and Huang (2011); and Sévi (2014), we use the decomposition of the quadratic variation with the intention of building a more accurate forecasting model. Our approach is very different though, as we use wavelets to decompose the integrated volatility into several investment horizons and jumps. Moreover, we employ recently proposed realized GARCH framework of Hansen, Huang, and Shek (2012). In contrast to popular HAR framework of Corsi (2009), realized GARCH allows to model jointly returns and realized measures of volatility, while key feature is a measurement equation that relates the realized measure to the conditional variance of returns. In addition, we benchmark our approach to several measures of realized volatility and jumps, namely realized volatility estimator proposed by Andersen, Bollerslev, Diebold, and Labys (2003), the bipower variation estimator of Barndorff-Nielsen and Shephard (2004), the median realized volatility of Andersen, Dobrev, and Schaumburg (2012), and finally jump wavelet two-scale realized variance (JWTSRV) estimator of Barunik and Vacha (2015) in the framework of Realized GARCH, and we find significant differences in volatility forecasts, while our JWTSRV estimator brings the largest improvement. We use Realized GARCH models of Hansen et al. (2012) as well as realized GAS of Huang, Wang, and Zhang (2014) based on the observation-driven estimation framework of generalized autoregressive score models to build a realized Jump-GARCH modeling strategy. In addition, we also utilize Realized GARCH with multiple realized measures (Hansen & Huang, 2012) to build a time-frequency model for forecasting volatility.

The main contribution of the paper is thus threefold.[4] First, we propose several model extensions to utilize jumps in the popular Realized GARCH frameworks, as well as build time-frequency model for forecasting volatility. Second, we use several popular measures as a benchmark to our time-frequency model. Third, we bring interesting empirical comparison of all frameworks in multiple-period-ahead forecasting exercise. We show that the most important information influencing the future volatility in foreign exchange is carried by the high frequency part of the spectra representing very short investment horizons. This decomposition gives us an interesting insight into the volatility process. Our newly proposed time-frequency models and Jump-GARCH models outperforms the existing modeling strategies significantly.

2. Theoretical framework for time-frequency decomposition of realized volatility

While most time series models are naturally set in the time domain, wavelet transform help us to enrich the analysis of quadratic variation by the frequency domain. Traders of the foreign exchange markets are operating with heterogeneous expectations, ranging from minutes to days, or even weeks and months. Hence volatility dynamics should be understood not only in time but at different investment horizons as well. In this section, we introduce a multiscale estimator that will allow these features and is moreover able to separate the continuous part of the price process containing noise from the jump variation. We will briefly introduce general ideas of constructing the estimator, while for the details necessary to understand the derivation of the estimator using wavelet theory, we refer to Barunik and Vacha (2015). In addition, we introduce several other estimators commonly used in the literature, which will serve as a benchmarks to us in the empirical application.

In the analysis, we assume that the latent logarithmic asset price follows a standard jump-diffusion process contaminated with microstructure noise. Let y_t be the observed logarithmic prices evolving over $0 \le t \le T$, which will have two components; the latent, so-called "true log-price process", $dp_t = \mu_t dt + \sigma_t dW_t + \xi_t dq_t$, and zero mean *i.i.d.* microstructure noise, ϵ_t, with variance η^2. In a latent process, q_t is a Poisson process uncorrelated with W_t, and the magnitude of the jump, denoted as J_l, is controlled by factor $\xi_t \sim N(\bar{\xi}, \sigma_{\xi}^2)$. Thus, the observed price process is $y_t = p_t + \epsilon_t$.

The quadratic return variation over the interval $[t-h, t]$, for $0 \le h \le t \le T$ associated with the price process y_t can be naturally decomposed into two parts: integrated variance of the latent price process, $IV_{t,h}$ and jump variation $JV_{t,h}$

$$QV_{t,h} = \underbrace{\int_{t-h}^{t} \sigma_s^2 ds}_{IV_{t,h}} + \underbrace{\sum_{t-h \le l \le t} J_l^2}_{JV_{t,h}} \tag{1}$$

As detailed by Andersen, Bollerslev, Diebold, and Labys (2001) and Barndorff-Nielsen and Shephard (2002a), quadratic variation is a natural measure of variability in the logarithmic price process. A simple consistent estimator of the overall quadratic variation under the

[3] Our interest is in return variation, although models attempting to capture the prices directly may be of interest.

[4] Note that our research adds to recent operation research contributions using wavelets in denoising of financial data, specifically high frequency data research (Haven et al., 2012; Marroquı, Moreno et al., 2013; Sun & Meinl, 2012), literature developing volatility models (Date & Islyaev, 2015; Pun, Chung, & Wong, 2015), literature contributing to forecasting volatility (Charles, 2010; Christodoulakis, 2007; Sévi, 2014) and studying stock market returns (Buckley & Long, 2015; Doyle & Chen, 2013; Wang, Zhang, & Zhou, 2015; Yang & Bessler, 2008).