European Journal of Operational Research 203 (2010) 14–21

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Discrete Optimization

Solving multiobjective, multiconstraint knapsack problems using mathematical programming and evolutionary algorithms

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ARTICLE INFO

Article history:
Received 7 November 2007
Accepted 26 June 2009
Available online 1 July 2009

Keywords:
Branch and bound
Knapsack problem
Multiobjective
Evolutionary algorithms

ABSTRACT

In this paper, we solve instances of the multiobjective multiconstraint (or multidimensional) knapsack problem (MOMCKP) from the literature, with three objective functions and three constraints. We use exact as well as approximate algorithms. The exact algorithm is a properly modified version of the multicriteria branch and bound (MCBB) algorithm, which is further customized by suitable heuristics. Three branching heuristics and a more general purpose composite branching and construction heuristic are devised. Comparison is made to the published results from another exact algorithm, the adaptive ε-constraint method [Laumanns, M., Thiele, L., Zitzler, E., 2006. An efficient, adaptive parameter variation scheme for Metaheuristics based on the epsilon-constraint method. European Journal of Operational Research 169, 932–942], using the same data sets. Furthermore, the same problems are solved using standard multiobjective evolutionary algorithms (MOEA), namely, the SPEA2 and the NSGAII. The results from the exact case show that the branching heuristics greatly improve the performance of the MCBB algorithm, which becomes faster than the adaptive ε -constraint. Regarding the performance of the MOEA algorithms in the specific problems, SPEA2 outperforms NSGAII in the degree of approximation of the Pareto front, as measured by the coverage metric (especially for the largest instance).

1. Introduction

The knapsack problem is a widely-studied combinatorial optimization problem that has applications in many fields (Martello and Toth, 1990). Mathematical Programming, Dynamic Programming and Metaheuristics are the most common tools for solving such problems. In the last decade, the multicriteria formulation of the knapsack problem (multiobjective knapsack problem, MOKP) and the construction of the corresponding Pareto front have attracted significant attention from the Operational Research and the Computational Science community. In the present paper we will deal with the most complicated case where multiple constraints are present, giving rise to the multiobjective multiconstraint knapsack problems – MOMCKP (Jaszkiewicz, 2004; Erlebach et al., 2002; Zitzler and Thiele, 1999; Klamroth and Wiecek, 2000). In the previous definition, the term "multiconstraint" may also be found as "multidimensional". Specifically, we are interested in solving:

$$\begin{aligned} &\max\ Px \quad \text{st } Wx \le c,\\ &x = (x_1, \dots, x_n)^T \in \{0,1\}^n, \\ &P \in \mathbb{R}^{k\times n}, \quad W \in \mathbb{R}^{m\times n}, \quad c = (c_1, \dots, c_m)^T \in \mathbb{R}^m. \end{aligned} \tag{1}$$

A solution x' is Pareto optimal (nondominated, efficient) if and only if it is feasible and there is no other feasible x such that $p_i x \geqslant p_i x'$ for $i = 1,2,\dots,k$ with at least one strict inequality. The set of the Pareto optimal solutions is coined as the *Pareto set* (in the decision variable space). In the case of MOMCKP it is actually the set of the nondominated binary vectors x whose corresponding images Px into R^k comprise the *Pareto front*(in the criteria space). Multiple constraints and multiple objectives are degenerated to the conventional knapsack problem if the W and P matrices are simply n dimensional line-vectors, namely, if $m = 1$ and $k = 1$, accordingly. In this paper, we deal with the case where $k = 3$, $m = 3$ and n is varying from 10 to 50 according to the data sets available in the literature (Laumanns et al., 2005, 2006).

We solve the problem exactly as well as approximately. Although the approximate solution of multiobjective combinatorial problems using metaheuristics is the main trend nowadays, the usefulness of exact algorithms is also undoubted. One of the basic reasons is the necessity for benchmarks for the approximate algorithms. The quality of the Pareto front approximation cannot be evaluated properly if the Pareto front is not exhaustively

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0377-2217/$ - see front matter
doi:10.1016/j.ejor.2009.06.024

computed. The generation of the complete Pareto front cannot be done without the use of exact algorithms.

In this work we suggest that the multicriteria branch and bound (MCBB) algorithm (Mavrotas and Diakoulaki, 1998, 2005) which was initially developed for mixed integer Multiobjective Linear Programming (MIMOLP) problems can be properly applied to solve multiobjective combinatorial optimization (MOCO) problems and specifically MOMCKP problems. For this purpose, we devise three branching heuristics and one composite branching and construction heuristic that are integrated in MCBB and essentially accelerate it. We compare our results to the published results that we found in the literature, specifically, the results of the adaptive ε-constraint method (Laumanns et al., 2005, 2006).

Moreover, we apply two well known multiobjective evolutionary algorithms (MOEA), namely NSGAII and SPEA2, on the same instances of the MOMCKP problems. The purpose is to examine the behaviour of mathematical programming methods along with standard metaheuristics that provide approximate solutions. Plots of the fronts are made and the coverage metric is computed for the approximate fronts, while a comparison between the performances of the two MOEA is performed.

The rest of the paper is organized as follows. Section 2 reviews the related literature while Section 3 presents the heuristics which were developed to accelerate the MCBB algorithm for the MOMCKP problem. Section 4 briefly discusses the evolutionary techniques. Section 5 presents the datasets and discusses the results along with the plots and the coverage metric statistics of computed fronts. Finally, Section 6 draws the main conclusions.

2. Related literature

The case of MOKP is well studied in the literature and various algorithms have been proposed. Basically, there are two variants of the MOKP, namely MOMCKP and multiobjective single-constraint knapsack problem (MOSCKP). Both variants of the MOKP, either with many constraints or with a single constraint, are traditional benchmarks within MOCO. Although our work focuses on MOMCKP, we review the literature of MOSCKP, for the completeness of the presentation. It must be noted that the great majority of the papers in the next paragraph refer to the bi-criteria case.

Regarding MOSCKP, which has attracted more attention in the Operational Research community, Gandibleux and Freville (2000) used a two-phases branch and bound method (Visée et al., 1998) as well as a bi-objective tabu search in order to solve bi-objective knapsack instances with one constraint. Teghem et al. (2000) have proposed an interactive multiobjective simulated annealing procedure for MOSCKP (with four objectives and one constraint). Klamroth and Wiecek (2000) discuss dynamic programming approaches for the MOSCKP and extend their analysis to MOMCKP through didactic examples (not providing simulations). Captivo et al. (2003) have introduced a reformulation of MOSCKP to a multicriteria shortest path problem over large networks, solving the resulting problem by a novel labeling algorithm. They provide only bi-objective results, due to the large acyclic networks inherent in the reformulation, that require significant computer memory. Also, single-constraint multiobjective (actually bi-objective) knapsack problems are addressed by the authors. Bazgan et al. (2009a,b) implement a dynamic programming algorithm based on dominance relations and item order for the exact solution of MOSCKP and present experimental results of improved performance compared to the methods of Captivo et al. (2003) and an Integer Programming based conventional epsilon-constraint method. The authors focus exclusively on the MOSCKP solving instances of up to 4000 items for the bi-objective case and up to 110 items for the three-objective case.

Zhang and Ong (2004) used an LP-based heuristic for generating approximations of the Pareto set in large MOSCKP instances. Scatter search is also used for large instances of MOSCKP providing approximations of the Pareto set (Gomes da Silva et al., 2006, 2007a). Lately, the same authors also used the core concept (that is widely used in single objective knapsack problems) in bi-criteria knapsack problems (Gomes da Silva et al., 2007b).

MOMCKP has attracted more attention in the Evolutionary Computation community. An early influential work is (Zitzler, 1999; Zitzler and Thiele, 1999), in which they provide approximations of the Pareto front for two, three and four criteria instances of the MOMCKP with 250 up to 750 items using the SPEA algorithm (the number of constraints was equal to the number of objectives). Jaszkiewicz (2002) evaluated the multiobjective genetic local search algorithm (MOGLS) in large instances of the MOMCKP. Comparison involved three other MOEA and a proposed MOGLS by the author. Objectives and constraints were 2–4 and items 250–750. The population size was 150–350 and the size of the temporary elite population used by MOGLS was 20, while the number of generations was 500. Jaszkiewicz (2004) performed a computational experiment involving 16 datasets and three MOEA (SPEA, NSGA and Pareto Memetic Algorithm, PMA) on the MOMCKP, and the population size was restricted to either 20 or 50 individuals. Objectives and constraints were 2–5 and items 100–750. Ten replications per case were made. The author admits that using larger population sizes results in longer running times and better quality of solutions (Jaszkiewicz, 2004, p. 426). Li et al. (2004) have proposed an estimation of distribution algorithm for MOMCKP and tested their method with datasets from Zitzler and Thiele (1999) in comparison to MOGLS of Jaszkiewicz (2002). Kumar and Banerjee (2006) provide a theoretical analysis on the running time of a novel MOEA on MOMCKP and provide some simulation results for datasets from Zitzler and Thiele (1999). Recently, Alves and Almeida combined the Tchebyscheff metric within a genetic algorithm in their MOTGA method (2007).

The only work found in the literature that solves MOMCKP exactly is Laumanns et al. (2006, 2005). This article may serve as a benchmark for three criteria MOMCKP instances, which are solved by the authors exactly as well as approximately. Nevertheless, the exact generation of the entire Pareto set in three objectives and three constraints imposes a severe bound on the number of items which can be solved, ranging from only 10 to 50 items, while bigger problems with 100 items are hardly solved (solution time more than 10 days).

Research combining MOSCKP and MOMCKP is more scarce. Gomes da Silva et al. (2004, 2006, 2007a) have used a scatter search method for bi-objective multidimensional knapsack problems, which were reformulated as single-dimensional knapsack problems, through Langrangean Relaxation. The authors focused on bi-objective problems with as many as one hundred constraints. Erlebach et al. (2002) provided a polynomial time approximation scheme based on linear programming for MOMCKP and a fully polynomial time approximation scheme for MOSCKP.

Shukla and Deb (2007) present a detailed comparative study of four representative classical Multiple Criteria Decision Making (MCDM) generating methods to one MOEA, namely NSGAII, in several bi- and three-objective nonlinear test functions of varying difficulty. This is one of the few works accommodating MCDM and evolutionary algorithms in Multiobjective Optimization, though in nonlinear unconstrained optimization. Finally, Zitzler et al. (2004) present a tutorial in Evolutionary Multiobjective Optimization (EMO) proposing the usage of a given Interface, named PISA, in future computational experiments in Multiobjective Optimization. According to the best of our knowledge, nevertheless, only one study has used PISA to MOCO problems to date (López-Ibáñez et al., 2006).