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Interfaces with Other Disciplines

A multiplier bound approach to assess relative efficiency in DEA without slacks

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In this paper, we propose a new approach to deal with the non-zero slacks in data envelopment analysis (DEA) assessments that is based on restricting the multipliers in the dual multiplier formulation of the used DEA model. It guarantees strictly positive weights, which ensures reference points on the Pareto-efficient frontier, and consequently, zero slacks. We follow a two-step procedure which, after specifying some weight bounds, results in an "Assurance Region"-type model that will be used in the assessment of the efficiency. The specification of these bounds is based on a selection criterion among the optimal solutions for the multipliers of the unbounded DEA models that tries to avoid the extreme dissimilarity between the weights that is often found in DEA applications. The models developed do not have infeasibility problems and we do not have problems with the alternate optima in the choice of weights that is made. To use our multiplier bound approach we do not need a priori information about substitutions between inputs and outputs, and it is not required the existence of full dimensional efficient facets on the frontier either, as is the case of other existing approaches that address this problem.

1. Introduction

In most cases in practice, the DEA models assess the efficiency of the inefficient units by using reference points on the frontier of the production possibility set (PPS) that are not Pareto-efficient. This happens as a result of the fact that these models usually yield zero weights for the optimal multipliers, or equivalently (by duality), strictly positive values for the optimal slacks, which means that the efficiency scores obtained for these units do not account for all sources of inefficiency. Bessent et al. (1988) deal with the so-called "not naturally enveloped inefficient units", which are defined as those that have a mix of inputs and/or outputs which is different from that of any other point on the efficient frontier. These authors report the results corresponding to several studies that reveal the high frequency of the not naturally enveloped inefficiency units in practice. These units are actually those in $F \cup NF$ according to the classification of the decision making units (DMUs) in Charnes et al. (1991) (the DMUs in F are on the weak efficient frontier whereas those in NF are projected onto points in F). It has been paid much attention in the literature to this type of DMUs, where we can find a wide variety of approaches intended to provide efficiency scores for them trying to avoid the problems with the non-zero slacks.

In this paper we propose a new approach to address this problem that deals with the dual multiplier formulation of the DEA models. This approach is based on imposing restrictions on the weights, which have been frequently used to obtaining non-zero weights, and also allow incorporating value judgements into the analysis (see Allen et al. (1997) and Thanassoulis et al. (2004) for a general discussion). The key issue when using weight restrictions is the setting of the bounds to be located in these constraints. We determine some weight bounds as the result of using an ancillary criterion of selection that makes it possible a specific choice of non-zero weights among the optimal solutions of the unbounded DEA models. We guarantee that the resulting DEA formulations provide efficiency scores that reflect the comparison of the unit under assessment with reference points on the Pareto-efficient frontier, which ensures zero slacks.

We can find in the literature different approaches to setting bounds in weight restrictions of DEA models. Weight bounds can be obtained by resorting to the opinion of some experts involved in the underlying production process, as in Beasley (1990) or in Takamura and Tone (2003). In this latter case, the authors process the information from experts by using additionally AHP (Analytic Hierarchy Process). We can also use the information on prices and/or costs as in Thompson et al. (1995, 1996). Or we can even combine expert opinion and price information as in Thompson et al. (1990, 1992). Other authors have proposed to use the optimal weights of some units considered as model DMUs in order to specify the bounds in the weight restrictions (see Charnes et al. (1990) and Brockett et al. (1997)). These approaches have been mainly used with either Cone Ratio (CR) models (Charnes et al. (1990)) or Assurance Regions (AR) models (Thompson et al. (1986)), which were originally developed with the purpose of incorporating value judgements into the analysis, i.e., prior information and/or

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accepted beliefs or preferences concerning the underlying process of assessing efficiency and, in addition, they often lead to non-zero weights. However, it often happens in practice that we do not have available either expert opinion or prices/costs information. In that case, there are different methods that can be of help for the estimation of weight bounds, and these obviously rely on the information provided by the data. Most of them are based on somehow handling the optimal weights of the unbounded DEA models. For example, once the unbounded DEA model is solved in a first stage and we have compiled a weight matrix for all the variables, Roll et al. (1991) and Roll and Golany (1993) claim that we can set the bounds: (1) after eliminating the outliers and the extreme weights, (2) by imposing that a certain percentage of weights falls within the bounds or (3) at an acceptable ratio of variation for each weight within the range of the unbounded weights. As acknowledged by these authors, one of the main difficulties with these techniques is that the bounds obtained may vary depending on the chosen solution among the alternate optima for the weights. There are also some approaches that are based on imposing some specifically developed lower bounds on the multipliers in the dual formulation of the used DEA model. The idea behind these approaches is obviously to have strictly positive weights and, consequently, by duality, zero slacks. In fact, the use of the well-known non-archimedean ε in this formulation leads to non-zero weights, but this model does not produce efficiency scores that can be readily used. To avoid this, Chang and Guh (1991) propose to replace this ε with a specific lower bound that is obtained from the smallest non-zero value of the multipliers for all the variables of the unbounded DEA model. The main difficulty with this approach is that the resulting model may become infeasible (Chen et al. (2003) show how to modify this bound to avoid the infeasibility problems), together with the fact that the bounds are obtained following a procedure that does not consider the possible existence of alternate optima for the weights. Thus, different optimal solutions may lead to different bounds and these may lead to different efficiency scores for the units under assessment. Chen et al. (2003) develop an alternative multiplier bound approach in which the lower bounds are determined by strong complementary slackness condition (SCSC) solution pairs for extreme efficient DMUs. However, as acknowledged by the authors, the results obtained may vary depending on the SCSC solution that is chosen. We also include here the approach in Dyson and Thanassoulis (1988) which, in the case of having a single input (or a single output), provides lower bounds for the output (input) weights by imposing the condition that it cannot be used less than some percentage of the average input level the DMU being assessed uses per unit of output, which is estimated by means of a regression analysis.

The existing work on facet models, which relates to the extension of the facets of the frontier, is in particular intended to address the problems with zero weights, and consequently with the slacks. Some of these approaches are based on the extension of full dimensional efficient facets (FDEFs) of the frontier. See Green et al. (1996), Olesen and Petersen (1996) and Portela and Thanassoulis (2006). In the two former, the authors define some new technologies from the FDEFs of the original frontier, whereas in the latter the proposed approach uses an AR model in which the bounds of the AR constraints are estimated from the marginal rates of substitution of these FDEFs. Obviously, these approaches guarantee non-zero weights, but they require the existence of such FDEFs on the frontier to be used, which does not happen very often in practice. See also Bessent et al. (1988), with the "constrained facet analysis" (CFA), for an algorithm along this line that proposes to project each inefficient unit onto an extended facet of the frontier that is suitably selected. CFA, however, guarantees neither that the facet wanted exists nor that the algorithm implemented finds this facet even if this exists. Lang et al. (1995) follow a similar idea and modify the previous approach by proposing to extend a facet of the PPS of a previously specified dimension, calling their approach "controlled envelopment analysis" (CEA). We note that CEA avoids the zero weights by imposing a non-archimedean ε used as lower bound for the multipliers in the CCR model without specifying how this value is to be implemented for use in practice.

Finally, it should be noted that the problem addressed here has also been approached in a different manner by dealing with the primal envelopment formulation of the DEA models. That is the case of the so-called "generalized efficiency measures" (GEMs) (see, e.g., Cooper et al. (1999), Pastor et al. (1999) and Tone (2001)), which are efficiency measures especially designed to account for both radial and non-radial inefficiencies, and so, avoiding the problems with the slacks.

We propose here another multiplier bound approach intended to provide efficiency scores for the inefficient units that account for all sources of inefficiency. It can be used when we do not have available either information on prices/costs or expert opinion reflecting value judgements related to the involved inputs and outputs, and also if FDEFs on the frontier do not exist. We develop a two-step procedure which, in a first step, is aimed at specifying some weight bounds, which are then used in the second step in the weight restrictions that are incorporated into the dual multiplier formulation of the used DEA model. The resulting models will provide us with the efficiency scores that are wanted. We will show that these models can be equivalently formulated as AR models. As for the specification of the weight bounds, these are determined on the basis of a choice among the optimal solutions for the multipliers of the extreme efficient DMUs in the unbounded DEA models that is made by using a selection criterion. In absence of information on the relative importance of the involved variables, this criterion seeks to avoid the extreme dissimilarity between the weights of the optimal DEA solutions that is often found in practice. Thus, we look for non-zero weights while at the same time we try to avoid large differences between the multipliers as much as possible. We would like also to stress both that the procedures proposed to setting weight bounds are not affected by the possible existence of alternate optima in the unbounded DEA models and that the models we develop do not have infeasibility problems.

The paper unfolds as follows: In Section 2 we develop the two-step procedure we propose to the setting of the bounds to be located in the weight restrictions of the DEA models used for the assessment of the inefficient units. Section 3 includes an illustrative example. In Section 4 we outline two possible extensions of the proposed approach. Section 5 concludes.

2. A two-step procedure to assess DMUs in $F \cup NF$ without slacks

The high frequency of non-zero slacks (or, equivalently, zero weights) when assessing inefficient units in practice is mainly due to the total weight flexibility implicitly used in DEA. As has been widely explained in the literature, this total weight flexibility often leads to unreasonable results, since the weights assigned to the variables considered are frequently inconsistent with the prior knowledge or accepted views on the relative value of the involved inputs and outputs. In particular, the DEA models, trying to show the units under assessment in their best possible light, usually exploit this total weight flexibility to the point of evaluating the DMUs by putting the weight solely on a few set of variables and ignoring the rest of the originally considered inputs and outputs by assigning them a zero weight. This also brings with it that we have strictly positive values for the optimal slacks when assessing the efficiency of the inefficient units, since these are projected onto the facets of the weak efficient frontier, which means that these units are evaluated with reference to points that are not