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An optimal solution technique to the single-vendor multi-buyer integrated inventory supply chain by incorporating some realistic factors

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ABSTRACT

This paper develops two generalized integrated inventory models to deliver a single product from a vendor to multiple buyers. To minimize the total cost of set up, ordering, inventory holding and transportation, the production flow is synchronized by transferring the lot with equal and/or unequal (either all are equal or all are unequal or a combination of equal and unequal) sized batches (sub-lots), each of which incurs a transportation cost. For easy implementation of the models, we relax some unrealistic assumptions in the existing models such as unlimited capacities of the transport equipment and buyers' storage, insignificant set up and transportation times, unlimited lead time and batch sizes. A common optimal solution technique to the models is derived and their performances are analyzed. Potential significances of the solution method are highlighted with solutions of some numerical problems. The importance of the relaxed factors and limitation of the models are discussed.

1. Introduction

Synchronization of the production flow of the vendor–buyer(s) integrated inventory system is essential for controlling the inventory in this system. Such synchronization can improve the profits of organizations considerably. Although researchers have given a considerable attention on synchronization of the single-vendor single-buyer integrated inventory system, the single-vendor multi-buyer integrated inventory case has gotten little attention in this regard. Besides, some unrealistic assumptions such as insignificant set up and transportation times, unlimited capacities of the transport equipment and buyers' storage, unrestricted lead time of supplying a batch and the boundless smallest batch size exist in most of the available models in both cases. Sometimes, set up of a machine takes a considerable amount of time because of testing and its readjustments. To satisfy demand of a product continuously, sum of the times of set up of a machine and processing of its first batch must be less than or equal to the available time of meeting demand by the previous lot after its processing being finished. Similarly, inspection, loading, delivery and unloading of a batch take a reasonable amount of time. To maintain continuous satisfaction of demand, either the previously used transport or a second transport must be ready in time at the vendor to transfer the next batch. Often, extra factors increase the time of transporting a batch. Higher capacities of the transport equipment and buyers' storage cost more. If they are not utilized properly, the concerned party has to spend extra money for their maintenance. Smaller lead time requires delivery of a product in smaller amount frequently leading to a higher transportation cost but smaller inventory cost. On the contrary, higher lead time requires delivery of a product in larger amount infrequently leading to a higher inventory cost but smaller transportation cost. In this case the product may become obsolescent due to the introduction of a modernized substitute. A good vendor works with a buyer closely to reduce lead time as much as possible down to a reasonable point, where it is acceptable to the buyer to maintain a stable production and delivery schedule (Monczka et al., 1988). So, we need a trade off between the two extremes. An obtained optimal solution to a developed model may restrict the smallest batch size to be less than one, an impractical situation, if it is not constrained to be greater than or equal to one. The vendor–buyer(s) integrated inventory models available in the literature had been developed based on some or all of these assumptions, and hence typically counter difficulties on the way of their implementations. Therefore, there exists a research scope on developing an efficiently synchronized generalized single-vendor multi-buyer integrated inventory model by taking into account the discussed factors.

Synchronization of the vendor–buyer(s) integrated inventory supply chain requires a close relationship among them (Goyal and Srinivasan, 1992; Thomas and Griffin, 1996; Martinich, 1997; Hill, 1999; Pan and Yang, 2002). But Joglekar and Tharthare (1990) argued that negotiated benefit sharing of the integrated system among the closely related parties is dependent of information sharing, communication, trust building, travel and executive times and hence never be costless. However, ignoring the cost of benefit sharing, researchers have given a lot of attention to the

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synchronization of the integrated production flow. With such an assumption a series of research has been carried out for the control of integrated inventory in processing single and multiple products in a multi-stage serial production system (Szendrovits, 1975, 1976, 1978; Goyal, 1976, 1977, 1978, 1979; Szendrovits and Drezner, 1980; Goyal and Szendrovits, 1986; El-Najdawi and Kleidorfer, 1993; Hoque and Kingsman, 1995, 2006). A constant lot size of an item (each of the items) is transferred between stages either only with equal or only with unequal or with equal and/or unequal sized batches. Assuming a costless way of benefit sharing, various kinds of synchronization have also been developed in the single-vendor single-buyer problem (Hill, 1997, 1999; Pan and Yang, 2002; Braglia and Zavanella, 2003; Ben-Daya and Hariga, 2004; Hoque and Goyal, 2006, 2009; Hill and Omar, 2006; Boute et al., 2007; Hoque, 2009; Sajadieh et al., 2009). Coordination of the single-vendor multi-buyer supply chain problem under stochastic demand by ignoring the cost of benefit sharing has received considerable attention of the researchers as well. In this stream of research Bernstein et al. (2006) dealt with simple pricing schemes while Sinha and Sarmah (2010) applied discount pricing strategy; Gürbüz et al. (2007) used centralized ordering policy that orders for all retailers simultaneously whereas Geng et al. (2010) analyzed both centralized and decentralized strategies; Li and Zhang (2008) and Helper et al. (2010) considered information sharing; Duan et al. (2010) proposed quantity discount incentive to retailers and Krichen et al. (2011) added delay payments incentive along with this; Darwish and Odah (2010) and Almehdawe and Martin (2010) employed vendor managed inventory mode of operation. Assuming deterministic demand and costless way of benefit sharing, the single-vendor multi-buyer integrated production flow has been synchronized by Hoque (2008), Zavanella and Zanoni (2009) and recently by Hoque (2011). Hoque (2008) transferred the lot only with equal or only with unequal sized batches and accumulating the inventory at the vendor and buyers individually. Zavanella and Zanoni (2009) transferred the lot only with equal sized batches and accumulating the inventory at the buyers. Hoque (2011) transferred the lot from a vendor to multiple buyers with l number of unequal sized batches first; where the next one is a multiple of the previous one by the ratio k (>1) of the production and the total demand rates, followed by (n-l) number of equal sized batches. The equal sized batches are restricted to be less than or equal to the lth batch (the largest unequal sized batch) multiplied by k. The models developed were solved by applying Lagrangian Multiplier method. However, in cases of single-vendor single-buyer or single-vendor multi-buyer or multi-stage production, synchronization of the production flow by transferring the lot with equal and/or unequal sized batches was found to lead to the least total cost for some numerical problems. Although Hoque (2011) served that purpose, it did not cope with the relaxation of the discussed impractical assumptions. Besides, there arise mathematical complexities because of the imposed restriction on equal sized batches and the solution method used. Relaxation of the discussed assumptions would impose more constraints, and hence the complexities will be intensified if Hoque (2011) is extended. To get rid of this burden, we synchronize the production flow by transferring the lot with the same type of unequal sized batches as in Hoque (2011), but restricting the equal sized batches to be equal to the lth unequal sized batch (here the eth batch). Following this trend of synchronization, here we develop two generalized single-vendor multi-buyer integrated inventory models by accumulating the inventory at the vendor and buyers independently, but with the traditional trend of ignoring the cost of benefit sharing. Transportation of each of the batches incurs a transportation cost. In order to implement the models by taking into account the industry reality, we also incorporate them with the relaxation of the discussed impractical assumptions. Then we present a common minimal cost solution technique to the models by applying the general method of differentiation. Thereafter, comparative studies of the solution technique with the existing ones on numerical problems are carried out to show its potential significance. Next we illustrate the solution method including the relaxed factors with a numerical problem. To show the generalization and limitation of our models, a comparative study of the methods on the results of the single-vendor single-buyer numerical problems, originally solved by Hill (1999) and Hill and Omar (2006) is carried out. Finally, we discuss the importance of the relaxed assumptions and highlight the scope of future research on the topic.

The outline of the remainder of the paper is as follows: Section 2 deals with assumptions and notations, and then presentation of the models and their optimal solution techniques. This follows a section of solutions of numerical problems and comparative studies of the methods both on the single-vendor multi-buyer and the single-vendor single-buyer numerical problems. The last section finishes with discussion and conclusion.

2. Models formulation

2.1. Assumptions and notations

In developing the models we assume

(i) Deterministic constant demand and production rates;
(ii) No backlogging or deliberate planning for shortages;
(iii) Both the vendor and the buyers agree to share the benefit of the integrated inventory system through negotiation but without incurring any cost.

We use the following notations in developing the models:
For the vendor

D Annual rate of demand; P Annual rate of production ($P > D$ and $k = P/D$);
h Inventory carrying cost per item per year; S Production set up cost per lot;
$s1$ Set up time (in yr); z The smallest batch (part of a lot) size;
Q The lot size transferred from the vendor to the buyers;
n Total number of equal and/or unequal sized batches in a lot;
e Number of unequal sized batches in a lot;
L_h The largest lead time and L_s The smallest lead time
For the i*th buyer* ($i = 1,2,\dots,m$);
D_i Annual rate of demand $(D = \sum_{i=1}^{m} D_i)$;
h_i Inventory carrying cost per item per year; s_i Cost of placing an order;
T_i Cost of transporting a batch from the vendor;
$g_{i,t}$ Capacity of the transport vehicle;
$g_{i,s}$ Capacity of the buyer's storage ($g_{i,s} \geqslant g_{i,t}$);
$t_{i,1}$ Inspection, loading, transfer and unloading time (in year);
$t_{i,2}$Return time of the transport vehicle (in year); z_i The smallest batch size;

2.2. Model I (Assuming a batch transfer just after finishing its processing)

2.2.1. The total cost function

Assume that $z_i = D_iz/D, kz_i = D_ikz/D, \dots, k^{e-1}z_i = D_i\, k^{e-1}z/D$, that is

$$k^{j-1}z_i = D_ik^{j-1}z/D \quad \text{for } j = 1, 2, \dots e; \text{ so that } k^{j-1}z = k^{j-1}\sum_{i=1}^{m} z_i.$$

Let the vendor transfers the first batch of size z to the buyers. Note that $z_i = z_i \Rightarrow D_iz/D = z_i \Rightarrow (P/D)(D_iz/P) = z_i \Rightarrow kz/P = z_i/D_i = z/D$.