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Discrete-time, economic lot scheduling problem on multiple, non-identical production lines

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ABSTRACT

This paper deals with a general discrete time dynamic demand model to solve real time resource allocation and lot-sizing problems in a multimachine environment. In particular, the problem of apportioning item production to distinct manufacturing lines with different costs (production, setup and inventory) and capabilities is considered. Three models with different cost definitions are introduced, and a set of algorithms able to handle all the problems are developed. The computational results show that the best of the developed approaches is able to handle problems with up to 10000 binary variables outperforming general-purpose solvers and other randomized approaches. The gap between lower and upper bound procedures is within 1.0% after about 500 seconds of CPU time on a 2.66 Ghz Intel Core2 PC.

1. Introduction and literature review

In industries such as plastics extrusion, metal stamping, textile manufacturing, bottling, printing and packing, it is a common practice to produce items on a capable production line using a single stage production process, such as molding or pressing. Typically, the manufacturing facility consists of many production lines or machines. Some of these lines are dedicated to the production of a single high-demand item (focused factory) while other lines each produce a set of medium to low-demand items (flexible factory). Usually, the different production lines have their specific characteristics such as setup or changeover times, finite production rates in each period (capacity limitations) and costs. For simplicity, we assume that all the production lines are physically co-located so that material transportation costs may be ignored. The relevant costs we consider include the fixed setup cost of producing a batch of each item, the item inventory carrying cost, the unit production cost for each item on each line and a shortage penalty cost for lost sales/penalty costs for backlogged demand. The main decisions to be made are:

1. *Resource Allocation*: Which items should be made on which lines?
2. *Lot Sizing*: How often and in what batch sizes should the items be produced?
3. *Scheduling*: How should item production be scheduled on each line?

Versions of this problem are widely prevalent in practice, however, adequate solutions are not, and past research has mostly addressed special cases. As we shall see, the problem is non-trivial and good answers to the above questions have the potential of generating significant operational cost savings. Further, what-if sensitivity analysis of the problem may also be used in making strategic decisions involving equipment selection for capacity expansion, and for tactical resource utilization and maintenance planning.

The problem of batching and sequencing the production of items on a *single* line, given a constant demand rate to be satisfied by this line, has been traditionally classified as the economic lot scheduling problem (ELSP). The traditional ELSP problem is based on continuous time case (constant demand rate) and has been extensively studied, see Elmagraby (1978), Muckstadt and Roundy (1993), Potts and Van Wassenhove (1992), Pinedo (2002), Pinedo (2005), Zipkin (2000) for a review. Jones and Inman (1989) demonstrate that the simple rotation cycle heuristic is often near-optimal. For the ELSP, Gallego (1993) and Gallego and Moon (1992) consider the effect of reducing setup costs/times and production rates respectively. For the ELSP problem with discrete time case (i.e. different demand for each period over a finite horizon of time), Dobson (1987) develops good feasible schedules with a more general production sequencing and time-varying lot sizes (also see Zipkin (1991) for computation of optimal lot sizes in this context). For a comprehensive review of ELSP problems (continuous and discrete), refer to Drexl and Kimms (1997) and Quadt and Kuhn (2008).

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This paper deals with Discrete-Time ELSP in a multi-machine environment. That is, each item requires processing on a single production line (machine or facility), however there are many lines that are capable of processing each item, at different production rates, with different unit production costs, setup costs and setup times. Our goal is to apportion item production to the different lines and generate batch sizes and schedules to minimize the total production, setup and inventory costs, plus the cost of lost sales/cost of backlogging, resulting from insufficient capacity, over a finite horizon of time, given the demands for different products in each time period.

The identical machines version of this problem, for a continuous time case, without lost sales, has been considered by Maxwell and Singh (1986) and Carreno (1990). Maxwell and Singh present conditions for the existence of feasible cyclic schedules with cycle length restricted to a power of k multiple of a base planning period. Based on a new mathematical formulation, Carreno develops a heuristic that first solves a capacitated assignment problem and then generates a rotation schedule consistent with the assignment of item production to machines. However, Carreno's formulation does not extend to the non-identical machines case. An extension to Carreno's work when some items cannot be produced concurrently has been addressed by Pesenti and Ukovich (2003).

Companies often have a combination of old or slow machines along with newer, faster machines. In practice, the shop floor controllers are routinely required to deploy these limited and non-identical resources efficiently. The research by Bollapragada and Rao (1999) addresses the multiple non-identical machine problem considered here for a continuous demand case, with an objective of minimizing the long-run average production, setup and inventory costs, plus the cost of lost sales resulting from insufficient capacity. Also, Bollapragada and Rao (1999) restrict attention to rotation cycles, and hence the paper addresses only the resource allocation and lot-sizing problem. The *research here* (for a discrete time demand case), is a first step in understanding how resource allocation, lot-sizing and scheduling interact and their impact on overall costs, for a finite horizon time, given the demands for different products in each time period. Hsu (1983) has proven that the Discrete-Time ELSP problem for a single machine is NP-hard, and hence this property applies to our problem (multimachine discrete time ELSP).

Further, in this research, we present three different variations of our deterministic demand model. In the first case, called model A, demands for all products must be satisfied in each time period. No lost sales or backlogging is permitted. In the second case, called model B, excess demand for products in each time period is lost with a penalty cost. This corresponds to an environment where the production exceeds the demands in certain periods, and the customer is unwilling to wait into another period. In the third case, called model C, excess demand for products in each time period is backlogged to future periods, with a penalty cost. This corresponds to an environment where the production exceeds the demands in certain periods, and the customer is willing to wait into another period.

Under the above three circumstances, we want to minimize the overall costs (production, inventory, setup, lost sales/backlogging costs) over a finite horizon of time.

The paper is organized as follows. In Section 2 the problem and the models are introduced. Section 3 presents two constructive heuristics, while Section 4 describes a metaheuristic (based on the Recovering Beam Search approach) and an improvement procedure (based on a mat-heuristic method). Section 5 presents our computational testing.

2. A discrete time, finite horizon model

The research problem is to find an assignment of the item production to the different machines to meet the demand at the end of each time period, with an objective of minimizing the sum of the production, inventory and setup costs.

The notation used is as follows (where the terms item and product are used interchangeably):

P, M, T	= The number of products, production lines (machines) and time periods, respectively.
K_{ij}, S_{ij}	= Setup cost and setup time for product i on line j, respectively.
D_{it}	= Demand for product i at the end of period t.
T_{jt}	= Available productive time on line j in period t.
h_i	= The holding cost per unit per period for product i.
C_{ij}	= The unit production cost for item i on line j.
p_{ij}	= Processing time for item i on line j.
X_{ijt}	= Production quantity of item i on line j in period t.
Y_{ijt}	= 1 if item i is setup on line j in period t.
I_{it}	= Inventory of item i at the end of period t.

For all the three models presented below, we assume the starting inventories to be zero: $I_{i0} = 0, \forall i \in 1,\ldots,P$.

2.1. Model A

Let us define N_{ijt} as the maximum possible production of item i on machine j in time period t:

$$N_{ijt} = \frac{T_{jt} - S_{ij}}{p_{ij}}$$

The objective of model A is to minimize the sum of the production, inventory holding and setup costs. The integer programming model of the problem is then as follows:

$$\min \sum_t \sum_i \left(h_i I_{it} + \sum_j (K_{ij} Y_{ijt} + C_{ij} X_{ijt}) \right)$$

$$I_{it-1} + \sum_j X_{ijt} - I_{it} = D_{it} \quad \forall i, t$$

$$\sum_i (S_{ij} Y_{ijt} + p_{ij} X_{ijt}) \leqslant T_{jt} \quad \forall j, t$$

$$X_{ijt} \leqslant N_{ijt} Y_{ijt} \quad \forall i, j, t$$

$$X_{ijt} \geqslant 0, I_{it} \geqslant 0, Y_{ijt} \in \{0, 1\} \quad \forall i, j, t$$

The first and second constraints specify the inventory balance, and capacity constraints respectively. The third constraint sets limits on the value of the production quantity for item i in period j, in each time period.

In the above formulation, demand for each product in a time period, must be satisfied from either that period's production or inventory carried into that period. As no lost sales or backlogging is permitted, some demand may never be satisfied, for some problem instances. In such cases, the problem instance is infeasible.

2.2. Model B

We then consider a variation of this problem, where demand that cannot be filled from inventory or current production at the end of each time period is lost, with a lost sales cost of l_i, for item i. This corresponds to an environment where the cumulative demand exceeds the cumulative production capacity in certain time periods, and the customer is unwilling to wait into another time period. Let us define:

l_i= Lost sales cost for item i.
L_{it}= Lost demand for item i at the end of period t.

The new problem formulation for the lost sales case is model B, and is as follows: