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Lending decisions with limits on capital available: The polygamous marriage problem

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ABSTRACT

In order to stimulate or subdue the economy, banking regulators have sought to impose caps or floors on individual bank's lending to certain types of borrowers. This paper shows that the resultant decision problem for a bank of which potential borrower to accept is a variant of the marriage/secretary problem where one can accept several applicants. The paper solves the decision problem using dynamic programming. We give results on the form of the optimal lending problem and counter examples to further "reasonable" conjectures which do not hold in the general case. By solving numerical examples we show the potential loss of profit and the inconsistency in the lending decision that are caused by introducing floors and caps on lending. The paper also describes some other situations where the same decision occurs.

1. Introduction

Financial regulators in several countries have sought to rein in or alternatively encourage lending to borrowers by putting caps or floors on the amount a bank can lend over a given time period. Countries such as China, India or Turkey have imposed caps on lending for a number of years. More recently, the UK government also sought to have an annual floor on a bank's lending to small and medium sized enterprises (SMEs). Imposing such regulations on lending will change the decisions on whom to lend to as well as making the lending less profitable for the bank and in consequence leading to inefficient lending. It can also mean there is unfairness in the lending as the accept/reject decision by the bank depends on how much capital is still available before the limit is reached and how long it is until the end of the lending restriction horizon. The objective of this study therefore is to identify the impact of putting caps/floors on inefficiencies and unfairness by solving the lender's accept/reject decision problem optimally.

This paper develops a set of Markov Decision Processes (MDP) models which address the lending problem with constraints on the total capital lent. From these, it is possible to investigate the optimal lending policies and how they differ in which borrowers are being accepted and the lender's total profitability compared with the optimal policy when there is no restriction on capital. The models themselves have a flavour of multiple choice secretarial or marriage problems, or the house hunting problem but are quite different in the objective to be optimized and the information available to the decision maker. In this problem the objective is to maximise the total profit to the lender rather than maximising the probability of choosing the k most profitable borrowers. When a potential borrower requested a loan, the lender is told two characteristics – the size of the loan requested and the probability of the borrower not defaulting on the loan. The latter of these is given by a credit score. The form of the optimal policy is to accept a borrower if their probability of non-default is above a certain value which translates into the credit score being above some cut-off score. However, unlike the traditional problem with unlimited capital available, this cut-off level will vary depending on the capital still available and the time until the lending restrictions end. These models show how significant is this unfairness to borrowers and also the drop in the lender's profitability that these capital restrictions cause.

In Section 2, we review the literature outlining the restrictions on consumer lending, the basic consumer lending model and the related literature on the secretary problem. In Section 3 we define the lending model with a cap on the amount of capital that can be lent in a given time period. This is a Markov Decision Process model and we describe the optimality equation and the form of the optimal policy. We also suggest two other policies including the optimal policy if there is no cap on the capital. Section 4 describes a discrete state space simplification of the model. This allows us to calculate several numerical examples including some that prove to be counter examples to reasonable conjectures concerning the optimal policy.

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It also means we can evaluate the difference between the optimal capped and uncapped policies and so address the fairness and profit sub optimality of the former. Section 5 looks at the problem where there is a floor on the capital that must be lent. This has been advocated by several governments particularly for bank lending to SMEs. It turns out that this problem leads to essentially the same Markov Decision Process model as the lending problem with a cap on the capital. We write the original problem with general cost functions though we describe the problem in the text in terms of the lending problem. In Section 6 we describe three other problems which can lead to the model of Section 3. We draw some conclusions in the final section.

2. Literature review

In the last decade, regulators in several countries have sought to improve the economy by putting floors or caps on the annual level of an individual bank's lending to consumers or small businesses. The Central Bank of China set annual limits on the new lending by individual banks (Bloomberg, 2012) for a number of years. In 2011, Turkey's bank regulator penalised banks that exceeded a limit on consumer lending. On the other hand, some governments sought to increase the lending to companies, particularly small and medium sized (SMEs) ones by putting floors on each bank's lending in that sector. The most notable of these was the agreement between the UK government and four international banks enshrined in the Merlin Project (Bank of England, 2012). The Bank of India had already in 2006 imposed a floor on banks' lending in this sector (Reserve Bank of India, 2009).

The impacts of these restrictions on the individual lending decisions are modelled in this paper. Opportunities to invest arrive according to a random process and are described by two characteristics – the size of the resource required and the probability of the opportunity being successful. The distribution of these over the whole set of possible opportunities is known but not their value on a specific opportunity until it appears. If the opportunity is rejected it cannot subsequently be accepted. The objective is to maximise the total expected profitability of the opportunities accepted assuming that there is a fixed time horizon for investment and a limit on the resource available. This has the flavour of a number of classic decision problems but does differ from them in several aspects.

In the secretary problem, sometimes called the marriage problem and reviewed by Freeman (1983) and Ferguson (1989), each opportunity only has one characteristic whose probability distribution is not known. The objective is to maximise the probability of choosing the opportunity with the maximum characteristic value. There have been many variants of this problem including Smith (1975) who allows the opportunity to refuse to be accepted and Yoshidi (1984) who allows a change point in the probability distribution. Preater (1994) looked at the multiple-choice problem where one can choose k, where $k > 1$, opportunities and the objective is to maximise a utility function of the ranks of the k opportunities chosen. Bateni, Hajiaghayi, and Zadimoghaddam (2010) reviewed a number of multiple choice secretarial problems where the objective is to maximise the expectation of a submodular function based on the skills of the secretarial group chosen. Extension of the classical problem has also been done by Chun (1999) who looks at the decision problems when there are more than one choices are available or by Stein, Seale, and Rapoport (2003) who compares the computational complexity of three heuristic solution approaches.

In the house hunting problem (Ferguson & Klass, 2010), there is a cost of examining each opportunity. The value of the characteristic of each opportunity is i.i.d. and the distribution is known. The objective is to stop at an opportunity so the total value of the characteristic of that opportunity less the examination costs is maximised.

Choice problems closer to that considered in this paper are the sequential allocation problems first introduced by Derman, Lieberman, and Ross (1972). In these problems, opportunities arrive according to a random process over a finite time horizon. The decision maker has a limited amount of resource to invest and the profit from each opportunity invested in is the same non-decreasing function of the investment level. The difference with the problem considered in the following sections is that the amount to invest is a decision by the investor whereas with us it is a characteristic of the opportunity. Moreover, our opportunities have different profits since these depend on the probability of repayment of the opportunity. Prastacos (1983) extended this problem by allowing the profit to depend on the quality of the opportunity. However, the decision is still how much to invest rather than whether or not to invest the amount required, which is the case in our problem.

Another way of thinking about the problem is as a dynamic stochastic knapsack problem (Kleywegt & Papastavrou, 1998). In this problem, items arrive to be loaded on a container of fixed size (the resource). The size and value of each item is unknown until the item arrives although the distribution of sizes and values is known. There is a holding cost per unit time until it is decided to dispatch the shipment and there is a value for any unused capacity. The objective is to maximise the expected overall value dispatched less the costs. The decisions are whether to accept an item and when to dispatch the shipment. Setting the holding cost and the value of unused capacity to zero would lead to a problem similar to the simplest one considered in this paper. Kleywegt and Papastavrou (1998) look only at the case where all items are the same size, which is akin to the case in Theorem 2 of this paper. Kleywegt and Papastavrou (2001) looked at the case of variable sizes but when there is no deadline on when the shipment needs to be sent. They also looked at the problem with a shipment deadline and found under what conditions the optimal policy had some monotone properties, something that did not hold in general. Subsequent works (van Slyke & Young, 2000; Zhuang, Gumus, & Zhang, 2012) have modified the problem to deal with the conditions that arise in yield management. The difference to the problem considered in this paper is that our opportunities have a probability of success and so the optimal policy is based on a cut-off on the riskiness of the opportunity rather than a control limit policy on the size of the loan. Moreover, the boundary conditions of fully funding or partially funding the opportunities that edge over the constraint cannot be used in the knapsack approach. Similarly, the floor problem we discuss does not fit into the knapsack approach.

3. Lending model with ceiling on capital available

Opportunities arrive according to a Poisson process with arrival rate λ. Think of these as borrowers applying for a loan. Each opportunity i requires an investment of x_i and has a probability p_i of a successful outcome with a reward of S per unit invested if successful and a loss of $-F$ per unit invested if a failure. The expected reward is then $x_i(p_iS - (1 - p_i)F)$. For the lending problem to consumers, the model found in Thomas (2009) suggests

$$S = (r - r_F), \quad F = (l_D + r_F) \tag{1}$$

where r is the interest rate charged on the loan; r_F is the risk free interest rate at which the lender can borrow the money that is being subsequently lent; l_D is the loss given default on the loan, which is the percentage of the loan that is finally lost at the end of the collections process. The lender does not know in advance the required investment level or risk probability of each loan but does know that the overall distribution of (x, p) is given by a density function $f(x, p)$.

When the borrower arrives the lender finds out the x and p for that applicant. The latter is usually expressed as their credit score. The decision maker has to decide when each opportunity arrives whether to accept it or reject it but is only allowed to invest L in total in the time horizon T of interest. The aim is to maximise the expected reward in this period.