European Journal of Operational Research 249 (2016) 465–475

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Take it to the limit: Innovative CVaR applications to extreme credit risk measurement

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A R T I C L E I N F O

Article history:
Received 26 November 2013
Accepted 10 December 2014
Available online 19 December 2014

Keywords:
Uncertainty modeling
Credit risk
Conditional Value at Risk
Conditional probability of default
Capital buffers

A B S T R A C T

The Global Financial Crisis (GFC) demonstrated the devastating impact of extreme credit risk on global economic stability. We develop four credit models to better measure credit risk in extreme economic circumstances, by applying innovative Conditional Value at Risk (CVaR) techniques to structural models (called Xtreme-S), transition models (Xtreme-T), quantile regression models (Xtreme-Q), and the author's unique *i*Transition model (Xtreme-*i*) which incorporates industry factors into transition matrices. We find the Xtreme-S and Xtreme-Q models to be the most responsive to changing market conditions. The paper also demonstrates how the models can be used to determine capital buffers required to deal with extreme credit risk.

1. Introduction

The Global Financial Crisis (GFC) raised widespread spread concern about the ability of banks to accurately measure and provide for credit risk during extreme downturns. Prevailing widely used credit models were generally designed to predict credit risk on the basis of 'average' credit risks over time, or in the case of Value at Risk (VaR) models on the basis of risks falling below a pre-determined threshold at a selected level of confidence, such as 95 percent or 99 percent. The problem with these models is that they are not designed to measure the most extreme losses, i.e. those in the tail of the credit loss distribution. It is precisely during these extreme circumstances when firms are most likely to fail, and it is exactly these situations that the models in this study are designed to capture.

Although the use of VaR (which measures potential losses over a given time period at a pre-determined confidence) is widespread, particularly since its adaptation as a primary market risk measure in the Basel Accords, it is not without criticism. Critics include Standard and Poor's analysts (Samanta, Azarchs, & Hill, 2005) due to inconsistency of VaR application across institutions and lack of tail risk assessment. McAleer (2009) finds that the internal modeling VaR approach as contained in the Basel II Accord (now to be superseded by Basel III with a greater focus on cyclical risk) seemed to encourage excessive risk taking at the expense of providing accurate measures and forecasts of risk. VaR has also been criticized by Artzner, Delbaen, Eben and Heath (1999) as it does not satisfy mathematical properties such as subadditivity. Embrechts, Puccetti, Rüschendorf, Wang, and Beleraj (2014) summarize the weaknesses of VaR as being threefold. Firstly, it says nothing concerning the what-if question: "Given we encounter a high loss, what can be said about its magnitude?" Secondly, for high confidence levels, e.g. 95 percent and beyond, the statistical quantity VaR can only be estimated with considerable statistical and model uncertainty, i.e. forecasts can become more uncertain and unstable at higher confidence levels. Thirdly is the subadditive problem.

Conditional Value at Risk (CVaR) is a measure initially used in the insurance industry for determining extreme returns (those beyond VaR). The metric has been shown by Pflug (2000) to be a coherent risk measure without the undesirable properties exhibited by VaR. In terms of the three VaR shortfalls mentioned above by Embrechts et al. (2014), CVaR partly corrects the first problem in that it addresses losses of high magnitude, and corrects the subadditive problem. The authors state, in line with findings by McNeil, Frey, and Embrechts (2005), that the problem of being able to accurately estimate single risk measures at high confidence levels still remains. Kaut, Wallace, Vladimirou, and Zenios (2007) find that accuracy and stability of CVaR forecasts based on historical data can be impacted through mis-specification of the underlying distribution and through insufficient scenarios. CVaR has been applied to portfolio

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http://dx.doi.org/10.1016/j.ejor.2014.12.017
0377-2217/

optimization problems by Uryasev and Rockafellar (2000), Rockafeller and Uryasev (2002), Andersson, Mausser, Rosen, and Uryasev (2000), Alexander, Coleman, and Li (2003), Rockafellar, Uryasev, and Zabarankin (2006), Birbil, Frenk, Kaynar, and Noyan (2009), Menoncin (2009), Dupačová and Kopa (2014), and Mansini, Ogryzak, and Speranza (2014). CVaR has also been explored as a measure of sectoral market and credit risk by Allen and Powell (2009), Powell and Allen (2009), but compared to VaR, CVaR studies in a credit context are still in their infancy.

Given the importance of understanding and measuring extreme credit risk (which we define as the risk in tail of a credit risk distribution, i.e. that risk beyond a specified threshold such as the CVaR thresholds used in this article), the first aim of this study is to show how CVaR techniques can be applied to prevailing models to measure this tail risk, using a US dataset which includes 380 US companies, mixed between investment and speculative entities. Of course CVaR, by definition, must always be higher than VaR but the extent needs to be quantified to understand the level of risk that is being ignored by VaR measures and we do this as part of our first aim. The second aim is to show how the CVaR measures can be used by banks to measure capital buffers required to deal with volatility in credit risk. A link can be drawn between the volatility of the market asset values of banks (as measured by models like the Merton Distance to Default (DD) model explained in detail in this paper) and capital adequacy, as illustrated by the Bank of England (BOE, 2008). BOE reports that in 2008 UK banks had equity ratios of around 3.3 percent, and assuming volatility in market value of assets of 1.5 percent, this gives a Probability of Default (PD) of around 1 percent (per DD and PD equations (1) and (2)). If volatility doubles, then PD increases substantially to 15 percent. As bank PDs increase with deteriorating market conditions, so too does the chance of the assets needing to be liquidated at market prices. Therefore as PDs rose during the GFC, market participants changed the way they assessed underlying bank assets, placing a greater weight on mark to market asset values, implying lower asset values and higher potential capital needs for banks. Thus BOE sees the mark to market approach of a bank's assets as providing a measure of how much capital needs to be raised to restore market confidence in the bank's capitalization. Other prominent bodies who have promoted monitoring the DD of banks include the European Central Bank (ECB) who sees a reducing DD as a useful measure of bank distress, and the International Monetary Fund (Otsu, 2010) who sees Distance to Default in a bank context as "Distance to Capital" (DC), which indicates when capital has been eroded and needs to be restored. In line with this thinking by the BOE, ECB and IMF, we use the CVaR based volatility metrics in this study to determine what capital buffers are required to restore market confidence in volatile times. This focuses on capital buffers is consistent with Basel III capital adequacy requirements (Bank for International Settlements, 2012), whereby banks are required to hold countercyclical capital buffers to protect them in downturn times.

To ensure a thorough examination of CVaR metrics we use a range of models (four in total), as well as apply two techniques (Historical and Monte Carlo Simulation) to each model. The Monte Carlo method generates multiple random scenarios, with the key advantage being that thousands of potential scenarios can be generated and considered, as opposed to just a few discrete observations. This is especially advantageous with CVaR, where historical observations are only limited to a small number of observations in the tail of the distribution.

The third aim of this study is to ascertain which of the models are best able to measure credit risk in the different economic circumstances of the pre-GFC, GFC and post-GFC periods by correlating the model outputs with actual measures of credit risk, including Credit Default Swap (CDS) spreads, delinquent loans and charge-offs.

Our four models are based around CVaR type modifications to some of the most widely used existing credit models. The Merton (1974) structural model uses a combination of asset value fluctuations and balance sheet characteristics to measure Distance to Default (DD) and Probability of Default (PD). Moody's KMV model (with KMV standing for Kealhofer, McQuown and Vasicek, a credit analysis business acquired by Moody's) is a modified version of the Merton model (with modifications summarized in Section 2.2). Moody's KMV (2010) report use of their products by more than 2000 leading financial institutions in over 80 countries, including most of the 100 largest financial institutions in the world. Auvray and Brossard (2012) report that DD can be a good lead indicator of bank distress, due to its market component, when closely monitored by the shareholders. Our first model (Xtreme-S) applies CVaR techniques to this structural model by measuring the tail asset value fluctuations (those beyond VaR). Our second model (Xtreme-Q) applies quantile regression to the Merton structural model, by dividing the dataset of asset value fluctuations into parts (quantiles), allowing the selected quantile (in our case based on tail observations) to be isolated and measured. Our third model (Xtreme-T) applies CVaR techniques to the CreditMetrics Transition model, which measures VaR and is the credit equivalent of the RiskMetrics model of Morgan and Reuters (1996) who introduced and popularized VaR. The CreditMetrics model incorporates credit ratings and calculates VaR based on the probability of transitioning from one rating to another (including to a default rating). Traditionally, transition models have been primarily used to measure corporate credit risk, but have also been used to measure consumer credit risk (Malik & Thomas, 2012) and for even wider applications such as the spread of infectious disease (Yaesoubi & Cohen, 2011). Our fourth model (Xtreme-*i*) applies CVaR techniques to our own *i*Transition model which is a transition model modified to incorporate market derived sectoral risk weightings. Whilst these credit models all have different outputs (for example, VaR as compared to DD), this is not a major concern for our study as we are interested in relative changes in measurements in each of our selected periods, rather than absolute measures.

The remainder of the paper is structured as follows: Section 2 describes data and the methodology (both Historical and Monte Carlo) used for each of the four models; Section 3 discusses results and implications for capital adequacy; Section 4 concludes.

2. Data and methodology

2.1. Data

In order to examine our models over different economic circumstances, data is divided into three periods: pre-GFC, GFC, and post-GFC. For each of the four models we generate separate measurements for each of these three periods. We also generate an annual measure for each model for each of the 13 years in the dataset. Our pre-GFC period includes the 7 years from 2000 to 2006. US banks were not regulated according to Basel Accord advanced model credit risk requirements at this time, but in terms of providing a useful benchmark, we note that this 7 year period aligns with the Basel Accord advanced model credit risk requirements. Our GFC period includes 2007–2009 which was the height of the GFC, and the post-GFC period is 2010–2012. Although our pre-GFC period is longer than the other two periods, we have opted for periods of different economic circumstances rather than periods of equal length. The 7 years prior to 2007 were a period of growth, followed by a crisis period until 2009, followed by a period of recovery, and our split represents these three different sets of circumstances. We also checked to see if there was any major difference between using a 3 year pre-GFC period (2004–2006) and a 7 year pre-GFC period (2000–2006) and found no significant difference between the VaR, CVaR, DD and CDD outcomes for these two period lengths and so we retained the 7 year pre-GFC period.

For our Merton/KMV based models (Xtreme-S and Extreme-Q) which require equity prices, we obtain daily prices from Datastream (approximately 250 observations $\times$ 13 years $=$ 3250 observations per company). Required balance sheet data for the structural model, which includes asset and debt values, is also obtained from